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THE FOLLOWING PAGES, 1729-1844 of Proceedings V. 57, contain discussion of JOURNAL papers published January through March, 1961, in accordance with the plan for quarterly publication of discussion.

Those readers who assemble their own Proceedings volumes for binding should plan to remove these pages to add to Proceedings V. 57, which will be completed with Part 2, December 1961 JOURNAL, which contains contents, indexes, errata, and final quarterly installment of discussion.

Contents of the following pages are listed on the outside back cover of this JOURNAL.

Discussion of a paper by Alan H. Mattock and Ladislav B. Kriz:

Ultimate Strength of Nonrectangular Structural Concrete Members*

By A. AAS-JAKOBSEN, A. A. GVOZDEV, and AUTHORS

By A. AAS-JAKOBSEN†

The authors give an interesting analysis of 18 beam tests which were carried out at the PCA laboratories in 1959. The purpose of this contribution is to add a few supplementary remarks to that analysis.

It is assumed that the compression zone has a constant compressive stress $f_c = 0.85\, f_c'$. When subjected to a pure moment the carrying capacity of the section, at the point of yielding of the reinforcement, will be

$$M = A_s f_y d(1 - 0.67\sqrt{A_s f_y / f_c d^2})$$

with $q = A_s f_y / f_c d^2$ and $\mu = M/f_c d^3$, hence $\mu = q - 0.67q\sqrt{q}$.

To obtain the maximum carrying capacity of the compression zone, the section is assumed to be subject to a moment, $\mu f_c d^3$, and a normal compressive load, $\eta f_c d^2$. The cross section of the concrete member is divided into two halves and the concrete compressive stress is assumed to be constant in both these halves, $f_c = 0.85\, f_c'$ in the most stressed half and f_1 in the least stressed half (stepwise constant stress distribution‡). The condition for equilibrium of the cross section gives $\eta + \alpha\mu = 1 + q - \alpha q/3$, where $\alpha = (1 + 2q)/(0.195 + 0.67q)$.

With $\eta = 0$, the equation is valid for a pure moment, and the following equation for the determination of q is obtained, viz., $\mu = q - 0.67q\sqrt{q} = (1 + q)/\alpha - q/3$, from which $q = 0.38$ and the ultimate value of the moment M becomes $M_{ult} = 0.85\, f_c'\, d^3\, 0.38(1 - 0.67\sqrt{0.38}) = 0.19\, f_c' d^3$. When $d = 7$ in., $M_{ult} = 65\, f_c'$.

The ratio between measured and calculated ultimate moment for Series T-1–T-6 is shown in Table A. The beams in Series L-1–L-12 have one-sided compression flanges and will not primarily develop compression failures with the reinforcement in question. The moment at the

*ACI JOURNAL, *Proceedings* V. 57, No. 7, Jan. 1961, p. 737.
†Member American Concrete Institute, Consulting Engineer, Oslo, Norway.
‡Aas-Jakobsen, A., "Trinnvis konstant spenningsfordeling," *Nordisk Betong* (Oslo), No. 3, 1960.

TABLE A — RATIO BETWEEN MEASURED AND CALCULATED ULTIMATE MOMENT FOR BEAMS T-1—T-6

Beam No.	T-1	T-2	T-3	T-4	T-5	T-6	Mean value	Standard deviation
Mattock-Kriz	1.01	1.12	1.02	1.13	1.18	1.14	1.10	0.069
Aas-Jakobsen	1.02	1.06	1.01	1.14	1.12	1.01	1.06	0.058

TABLE B — RATIO BETWEEN MEASURED AND CALCULATED ULTIMATE MOMENT FOR BEAMS L-1–L-12

Beam No.	L-3	L-7	L-11	Mean value (12 beams)	Standard deviation (12 beams)
Mattock-Kriz	1.12	1.26	1.46	1.10	0.138
Aas-Jakobsen	1.03	1.04	1.17	1.05	0.055

yield point of the reinforcement is given by $M_s = A_s f_y (d - 0.105\, A_s f_y / f_c')$. The ratio between the measured and calculated ultimate moment for Beams L-3, L-7, and L-11 is shown in Table B.

By A. A. GVOZDEV*

A correct estimate of the strength of nonrectangular structural concrete members is of important practical significance but tests concerning this subject are scanty. The authors' article is a valuable contribution to our knowledge in this field.

The writer prefers the simplest design formulas which should, however, be satisfactory for practical purposes. The comparison of the two theories given by the authors seems to confirm this opinion.

The equivalent uniform stress distribution theory is in many respects similar but more conservative than the theory adopted for USSR structural regulations. First, the magnitude of the uniform compressive stress in concrete is assumed to be $0.85 f_c'$ while a value approximately equal to f_c' is adopted in the USSR code. The second and more important difference consists in the method of evaluating the strength controlled by compression. Here I am going to use the term "compression zone" in the sense of the area on which uniform compressive stresses are assumed to act in theory. For structural members with a triangular compression zone as well as for members with a rectangular one the following limitation is established in the USSR code: the static moment of the compression zone area with respect to the axis passing through the center of gravity of the main reinforcement does not exceed 80 percent of the static moment with respect to the same axis of the full concrete

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area situated over the reinforcement. When disregarding the safety factors, the condition of strength for structural members reinforced only on the tension side can be expressed by the formula, $M = S_c f_c'$, and the inequality

$$S_c \leq 0.8\, S_0 \qquad \text{(A36)}$$

where M is the ultimate moment of external forces with respect to the axis passing through the center of gravity of the reinforcement. S_c is the static moment of the compression zone area with respect to the same axis, and S_0 is the static moment with respect to the same axis of the concrete area lying on one side of this axis..

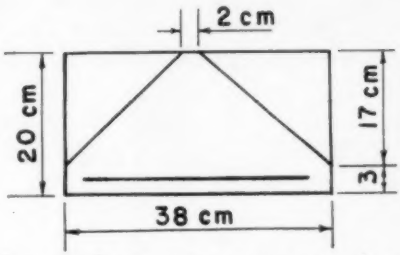

Fig. A—Beam cross section tested at Sverdlorsk

These conditions are applied also to structural members subject to bending and compression.

When the tension stress in the reinforcement reaches the yield point, the compression area is determined from the equilibrium equation. For members failing in compression before the yield point stress of the reinforcement is attained S_c is taken equal to $0.8\, S_0$ and the condition of strength becomes

$$M = 0.8\, S_0 f_c' \qquad \text{(A37)}$$

At first this design method was adapted for rectangular structural members. Then its application was extended to members of many other forms and especially to members with triangular compression zones.

I will tell briefly the reason why this was possible. In 1939, Bychkov in Sverdlovsk tested 12 beams[8] the middle part of which had an almost triangular cross section (see Fig. A) whereas the remaining parts of the beams were rectangular. The concrete strength of 20-cm cubes did not exceed 200 kg per sq cm, and the yield stress of the reinforcement did not exceed 3000 kg per sq cm. The reinforcement area in this series of beams was variable, which resulted in different types of failure; tension failure for eight beams and compression failure for the remainder. The results of control calculations of these beams according to the previously mentioned principles were as follows: the test value of the moment corresponding to the yield stresses in the reinforcement exceeded the calculated value by 4 percent on the average. For over-reinforced beams the test value of the ultimate moment was from 11 to 25 percent greater (18 percent on the average) than the computed values.

Torianik in Poltava[9] tested two groups of rectangular columns in compression at oblique eccentricity. For the subject under discussion the

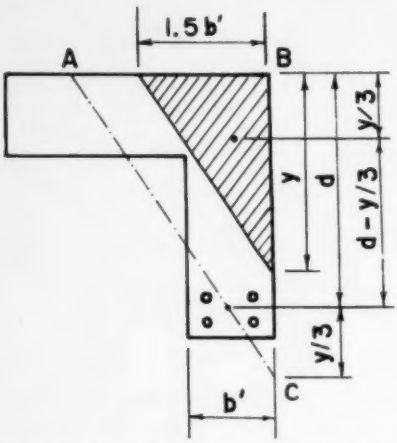

Fig. B—L-beam section with triangular compression zone

first test series completed in 1936-1937 is of prevailing interest. The eccentricity of loading for this series was great enough to provide a triangular, trapezoidal, or pentagonal compression zone. The cross-sectional dimensions were 25 x 30 cm, and the concrete strength was nearly 150 kg per sq cm. Twelve of the columns had a 1.5 percent reinforcement with a yield point stress of 2250 kg per sq cm. The reinforcement for one of the three remaining columns was half as great and for the last two columns greater than the former. The ultimate loads were computed as previously, but the reinforcing bars in compression were taken into account. The results were in good accordance with test data. The test strength was on an average 2.25 percent less than the calculated value, extreme deviations being +13 and −18 percent (in one case only).

In the laboratory headed by the writer, 13 columns of equilateral triangular cross section were tested by Bulgakov[10] to check the validity of Eq. (A36) and (A37). The triangle has 30-cm sides with 3.5-cm corners cut off. The area of the reinforcing bars situated near the corners was equal to 0.8 percent of the concrete area. Four columns were tested in simple compression. The eccentricity of loading for the remaining columns was chosen either in such a manner that at ultimate load the tension stress in the reinforcement should be somewhat less than the yield point stress or so that the yield stress should be reached immediately before the crushing of the concrete in compression.

For two of the columns according to the direction of eccentricity the shape of the compression zone was trapezoidal. For the seven remaining columns it was triangular. The cube strength of the concrete for the latter columns approximated 300 kg per sq cm. The test strength of this group of columns was on an average 3.3 percent less than the calculated value, extreme divergences being +21.4 and −23.1 percent. These results compare favorably with the theory.

Now I will try to apply the same methods of calculation to the test data of the authors and first of all to Series L. The authors showed conclusively that the compression zone of these beams must be triangular and that the usable width of the flange for all the beams is equal to 1.5 b'. Once this has been set it is easy to compute the strength of beams failing in tension. The calculation of strength for over-rein-

forced beams must be briefly explained. Although the cross section of the beams in this case is not triangular, the triangular compression zone permits the substitution of a fictitious triangular section ABC for the real one (see Fig. B). The length of the vertical side BC of the fictitious triangle must evidently be equal to $d + y/3$. Considering that the linear dimensions of a triangular compression zone whose static moment equals $0.8\ S_0$ must be 71 percent of the dimensions of the triangle corresponding to a static moment S_0, the following equation can be set up viz., $y = 0.71\,(d + y/3)$, from which $y = 0.925\,d$ and the lever arm of the inner forces is $0.69\,d$.

The ultimate moment can be now calculated by the formula, $M = (1/2)\ 1.5\ b' \times 0.925\ d \times 0.69\ d \times f_c' = 0.48\ b'd^2f_c'$.

The results of our calculations for Series L are summarized in Table C.

The agreement between test data and calculated values of ultimate moments is quite satisfactory, better even than in Table 8, especially for Beams L-7 and L-11 failing in compression. The analysis of Series T leads, however, to essentially different results.

The test strength is in all cases less than the theoretical values. The discrepancy grows with the concrete strength and becomes —20 percent for moderate concrete strength and —40 percent for concrete of high strength. The data of this test series are, apparently, not easily reconcilable with those from the USSR and even with those of Series L. In my mind, however, the discrepancies found for concrete of moderate strength can be considered to be accidental as the number of beams in Series T was small and because in Series L the strength of the heavily reinforced beam L-3 which had nearly the same range of concrete was sufficiently high. This cannot be said about beams with high strength concrete because the beams of Series L with comparable concrete quality failed in tension having a relatively weak reinforcement.

The strength of the heavily reinforced beams or eccentrically loaded columns with high quality concrete, if their compression zone is triangular, must be apparently evaluated more prudently. According to test results with rectangular beams and columns we reduce the usable stress of high strength concrete by introducing a

TABLE C—SUMMARY OF ULTIMATE MOMENT RESULTS FOR SERIES L

Beam No.	Ultimate moment, in.-kips		M_{test}
	M_{calc}	M_{test}	M_{calc}
L-1	664	678	1.02
L-2	721	693	0.96
L-3	1085	1062	0.98
L-4	1360	1325	0.98
L-5	652	671	1.03
L-6	743	778	1.05
L-7	890	876	0.98
L-8	1185	1173	0.99
L-9	655	663	1.01
L-10	686	757	1.10
L-11	1010	1129	1.12
L-12	1340	1265	0.94
		Average	1.015

special factor, the magnitude of which is less than unity. But these factors are not so low as those that one can deduce from the tests of Series T.

The investigation of the authors is a useful warning and must naturally be taken into consideration, though new and more numerous tests in the future would be desirable.

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9. Torianik, M. S., "Oblique Eccentrical Compression in Reinforced Concrete," *Gostekhizdat Ukrainy*, 1951. (in Russian).

10. Bulgakov, V. S., "Experimental Investigation of the Strength of Centrally and Eccentrically Loaded Concrete Columns of Triangular Cross Section," *Issledovanija po Teorii Zhelezobetona* (Moscow), V. 17, Scientific Institute for Plain and Reinforced Concrete of the Academy of Building and Architecture of the USSR, 1960, Gosstroiizdat. (in Russian).

AUTHORS' CLOSURE

The contribution of Professor Gvozdev provides welcome additional evidence as to the applicability to nonrectangular structural concrete members of fundamental plasticity concepts deduced from tests of rectangular members.

The use of a limiting static moment for the compression zone in the case of over-reinforced beams is equivalent to assuming a limiting neutral axis position for a particular cross section. If the rule quoted is applied to the case of a rectangular section it yields a maximum depth of the equivalent rectangular stress block of $0.55d$. This is close to the limiting value of $0.54d$ deduced by C. S. Whitney[11] from test results obtained by Slater and Lyse. Use of such an approach assumes that when ultimate strength is controlled by compression the depth of the neutral axis will be unaffected by concrete strength, or by the amount of reinforcement provided. A survey[12] of tests carried out on rectangular members showed that this is not the case. The authors are of the opinion that the discrepancies noted by Professor Gvozdev, between the strengths of the T series beams made from the higher strength concretes and the strengths calculated assuming a limiting neutral axis position, are because the neutral axis position at ultimate strength is not independent of the strength of the concrete. The dependence on concrete strength is taken into account in the theories detailed in the paper, with a resulting better agreement between the calculated and experimental ultimate strengths for this set of beams.

Dr. Aas-Jakobsen proposes a more sophisticated simplification of the actual curvilinear concrete compression stress distribution. As a result, he obtains calculated strengths more nearly in agreement with the test results than were obtained using the simple equivalent rectangular stress distribution. However, the strengths obtained by use of the equivalent rectangular stress distribution are close enough to the measured strengths for the purposes of practical design.

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12. Mattock, A. H., "The Strength of Singly Reinforced Beams in Bending," *Proceedings*, Symposium on the Strength of Concrete Structures, Cement and Concrete Association, London, May 1956.

Discussion of a paper by Bryant Mather and William O. Tynes:

Investigation of Compressive Strength of Molded Cylinders and Drilled Cores of Concrete*

By C. BERWANGER and AUTHORS

By C. BERWANGER*

The writer finds of considerable interest the study made by the authors. They confirm that the findings of Tucker,[4] regarding the variation of the standard deviation with specimen size, also apply to mass concrete. It would seem that there is not full agreement by researchers on the variation of the mean strength of a mix with specimen size, although the variations quoted seem to be small. No significant variation of mean strength with specimen size was found by the authors in their tests. However, the authors fail to discuss the variation of the standard deviation of mean strength with compressive strength level.

Neville[6] reported the results of a research that was conducted into the relation between the standard deviation of a mix and the mean strength of concrete test cubes. This paper includes a study of a large number of standard cube specimens and the standard deviations of each batch, as well as the limited number of cylinder strengths from another source.[7] The study includes the effects of age, with strengths at 3, 7, and 28 days included. The influence of cement type and the effect of the degree of control are also analyzed. In all of these comparisons it was found that there was a significant difference in the standard deviation as related to mean strength. It was found that the standard deviation of a mix increases directly with the mean strength.

To show that this variation exists in the test data of the paper, the writer, using the method of least squares, has fitted a straight-line equation to the test data of Table 1 for the core specimens. This relationship is as follows:

$$\text{Standard deviation} = 300 + 0.0128\ (\text{mean strength}) - 22.44\ (\text{core size})$$

Fig. A shows a plot of standard deviation versus mean strength for the values of Table 1 and for comparison the straight lines for the three core sizes are also shown. The cylinder test results are also plotted and show an increase of the standard deviation with mean strength. It can be seen that the straight-line approximations used are not the best possible representation of the relationship which is undoubtedly nonlinear. This is suggested by the curved trend of the standard deviation averages, disregarding core size, when mean strengths are less than 2000 psi.

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*ACI JOURNAL, *Proceedings* V. 57, No. 7, Jan. 1961, p. 767.

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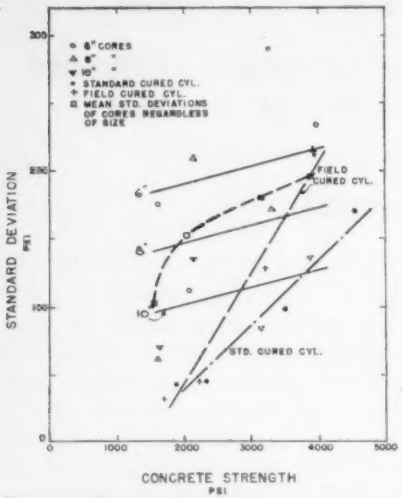

Fig. A—Standard deviation versus concrete strength

The object in presenting this analysis has been to show that factors other than specimen size affects variation of standard deviation. From the limited number of tests and the large scatter of results with respect to expressions developed, no general relationship is possible.

Aggregate size effect on mean strength cannot be determined from the test results; this would require mixes with 3- and 6-in. aggregates and similar W/C ratios.

The importance of the standard deviation, as related to concrete strengths, has been realized by researchers. In design it is also important. To insure a particular design strength, the actual strength must be determined. This can only be done if an estimate of standard deviation for the actual strength can be made for the particular fabrication conditions. The authors are to be complimented for presenting additional information on the standard deviation, as related to mass concrete.

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7. Bloem, D. L., "Studies of Uniformity of Compressive Strength Tests of Ready Mixed Concrete," *Bulletin* No. 206, American Society for Testing Materials, May 1955, pp. 65-70.

AUTHORS' CLOSURE

We appreciate Professor Berwanger's interest in the data given in our paper and we believe that their significance will be more fully realized when the variation of standard deviation of mean compressive strength with strength level is pointed out as he has done. We pointed out that the 28-day compressive strength is the property most frequently employed as an index of concrete quality. The usefulness of this index will be increased as its behavior becomes better understood by all who use it. We hope that our results, supplemented by the discussion given by Professor Berwanger, will not only contribute to a better understanding of the significance of some characteristics of the 28-day compressive strength as an index of concrete quality but will also encourage studies of other aspects of the behavior of this index.

Discussion of a paper by Paul Klieger and J. A. Hanson:

Freezing and Thawing Tests of Lightweight Aggregate Concrete*

By ROBERT K. DUEY, FRANK G. ERSKINE, D. W. LEWIS, and AUTHORS

By ROBERT K. DUEY†

The authors have presented some interesting data on freezing and thawing tests of lightweight aggregate concrete. This data has been vitally needed by the concrete industry as well as other consumer interests.

The writer believes that the data presented should be further examined and would like to offer several comments:

1. What correlation, in the interest of future test comparisons, can the authors offer between the freezing and thawing test method used in this test series with one of the current ASTM standards of test, particularly ASTM C 291-57T?

2. The use of a fine aggregate grading midway between the limits established in ASTM C 330-57T for Aggregates 2, 3, 4, 5, 6, and 7 may not have been fair to several of the aggregates involved. Most producers of lightweight aggregates have established by performance tests an optimum grading which they produce within certain tolerances, and this grading could differ markedly from the grading adopted in this test series. Specific gravity variations of the various sieve fractions of different lightweight aggregates would further indicate an arbitrary selection of a sieve analysis by weight for all the aggregates noted may not have been a fundamentally sound approach.

3. Do the authors have any explanation for the performance of Aggregate 6 which as tabulated in Table 5 showed poor resistance to freezing and thawing in non-air-entrained concrete but good resistance in air-entrained concrete exceeding the performance of several of the aggregates which exhibited a far greater resistance in non-air-entrained concrete?

4. The data in Table 5 also indicate that for the leanest concrete in this test series, the 3000-psi class, and non-air-entrained concrete, Aggre-

*ACI JOURNAL, *Proceedings* V. 57, No. 7, Jan. 1961, p. 779.
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gates 2, 3, 4, and 5 performed exceedingly well showing greater resistance than the Elgin sand and gravel in freezing and thawing tests.

5. It is felt that Paragraph 3 of the authors' summary could be misinterpreted. The test series included only two moisture conditions, namely 0.1 percent and moisture resulting from soaking the aggregates in water for 18 to 24 hr. Lightweight aggregates are seldom if ever soaked for this amount of time prior to batching. What would have been the results of these tests within normal practical limits of moisture from oven dry condition to saturation (1 to 2 hr water soaking)?

By FRANK G. ERSKINE*

This paper contains valuable and timely information for the concrete construction industry, particularly that segment concerned with bridge and grade separation structures subjected to severe climatic conditions. Laboratory evidence is presented which clearly indicates that structural lightweight concrete is capable of furnishing excellent resistance to freezing and thawing with air entrainment, and that fair to good resistance can be obtained without air entrainment with certain types of aggregates provided they are not saturated prior to mixing.

In recent years with the increased use of structural lightweight concrete it has been found that more efficient control of mixing and placing can be achieved with air entrainment and with the aggregates in a moist condition well below saturation. Many producers recommend that their aggregates be used with the moisture content rarely exceeding 50 percent of total absorption, and some recommend relatively dry material. In addition, the vast majority advocate the use of air entrainment.† These recommendations are consistent with the conclusions for obtaining durability in this paper.

The bulletin "Bridge Deck Survey," 1960 edition, published by the Expanded Shale Clay and Slate Institute lists over 150 bridges with structural lightweight concrete decks, more than half of which have lightweight concrete wearing surfaces. The service record of these structures is good and relatively few cases of poor performance have been reported. Recently, a field inspection was made of some 25 or 30 of these bridges including 2 or 3 on which poor performance had been reported. In each case of poor performance it was learned that the aggregates had been saturated prior to mixing and that there was little if any air entrainment in the concrete. These field observations confirm the conclusions of Messrs. Klieger and Hanson.

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† "Workability is Easy," 1958, and "Suggested Mix Design," 1958, published by the Expanded Shale Clay and Slate Institute.

A theory has been advanced that the air voids in structural lightweight aggregates, particularly if they are small, might offer escape space for freezing moisture. An examination of the performance of non-air-entrained air dried aggregate concretes in Fig. 3 and 4, seem to partly confirm this theory. On the other hand, regardless of the possible contribution of the pore structure to the concrete's durability, it is extremely satisfying to see the splendid performance of the air-entrained concretes as presented in Fig. 1 and 2.

Aside from the primary purpose of this investigation, the study contains much additional useful data. For example, Table 7 showing the resistance to deicer scaling is of particular value.

The authors are to be congratulated for a worthy contribution to the knowledge of the properties and performance of structural lightweight concrete.

By D. W. LEWIS*

The work reported by Messrs. Klieger and Hanson is a valuable contribution to the literature on lightweight concretes and aggregates and constitutes the first comprehensive study of the relative durabilities of lightweight and normal weight concretes in laboratory freezing and thawing. The results clearly indicate the importance of entrained air in producing durable concrete, and that lightweight concretes react in much the same way as normal weight concretes to the variables involved in the freezing and thawing test. The lack of correlation between concrete absorption and durability is valuable in supplying additional evidence that recent action deleting the absorption limits from ASTM C 330-60T was justified.

It is hoped that investigations of this nature will be continued and expanded, for many questions remain unanswered with respect to durability tests of lightweight concretes—and, in most cases, the same questions have yet to be satisfactorily resolved for normal weight concretes. These, of course, include the most appropriate type of freezing and thawing test, correlation of test results with field performance to permit selection of reasonable durability requirements, and proper moisture condition for the aggregates and curing for the specimens.

In the authors' tests, 14 days drying at 50 percent relative humidity was included in the curing period—a practice that is not ordinarily followed in tests of normal weight concrete. Whether this drying period is really necessary or desirable in testing lightweight concretes has not been adequately established. If it is needed, the effects of drying at

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different relative humidities may prevent comparison of test results between laboratories, since many organizations equipped for freezing and thawing tests do not have facilities for relative humidity control.

NSA TESTS

Results of a series of tests conducted on expanded slag concretes in the National Slag Association Laboratory* are of interest in this respect. Two expanded slags (air-dry at the time of mixing) were used in various mix combinations subjected to freezing and thawing. All concretes were air-entrained with 6.6 percent, the lowest air content used. No air drying period was used, instead the specimens were cured in the moist room for 26 days followed by 2 days immersion in water immediately prior to the first freezing cycle. Freezing was in air and the thawing in water. (This procedure is preferred by many investigators for evaluation of aggregates over the freezing in water employed by the authors, since it tends to produce less deterioration in the cement paste.)

Results for the NSA tests on 12 structural concrete mix combinations showed durability factors at 300 cycles of 99 to 108, quite comparable to the authors' results on air-entrained concretes made with dry expanded slag aggregate (Aggregate 6). Four insulating concrete mixes (lean, high water-cement ratio mixes with strengths from 650 to 1650 psi) had durability factors ranging from 78 to 100. In these tests, continuous moist curing did not adversely affect the durability.

The authors conclude that moisture condition of the aggregate was a significant factor in determining the freezing and thawing resistance of the concretes. While this conclusion is apparently justified for some of the materials tested, it is believed that moisture condition may not have exerted a direct influence in other cases. As the authors point out, all of the non-air-entrained concretes made with air-dried aggregates had greater durability than the corresponding concretes made with soaked aggregates. The data also show that in 14 of the 17 comparisons involving lightweight concretes, the mixes with air-dried aggregate had higher air contents than those with soaked aggregate. In two of the other three cases, differences in air contents shown were only 0.1 percent and durability differences may not be significant. Air contents of these non-air-entrained concretes are quite high in some cases (up to 4.5 percent for Aggregate 2 in the 3000-psi concrete).

The nature of the air, whether entrapped or entrained, and its effect on durability of the paste is unknown. The proportion of "effective" and "non-effective" air might also vary with the aggregate used. However, it seems probable that some of the air would be entrained (although not intentionally) and could control the durability to a large extent.

*Lewis, D. W., "Lightweight Concrete Made With Expanded Blast Furnace Slag," ACI JOURNAL, *Proceedings* V. 55, No. 5, Nov. 1958, pp. 619-633.

Such effects are likely to be particularly pronounced in tests involving freezing in water, and minor differences in air content may produce considerable variations in concrete durability.

DURABILITY FACTORS

The durability factors reported are a measure of the durability of the concrete in this particular type of exposure, but do not specifically indicate whether soaking the aggregate made it less durable, or whether the cement paste became less durable because of a lower content of unintentionally entrained air. For example, air-entrained mixes with Aggregate 2 had durability factors slightly higher for the soaked than for the dry aggregate in both the 3000- and 4500-psi concretes (a total range of 88-93). The non-air-entrained dry-aggregate mixes, with air contents of 4.5 and 3.0 percent for the 3000- and 4500-psi concretes, respectively, had durability factors of 85 and 86. Non-air-entrained soaked-aggregate mixes with 2.7 and 2.5 percent air had durability factors of 45 and 23. It might be concluded in this case that at least 3 percent air is required to adequately protect the cement paste in mixes with Aggregate 2, and that durability of the aggregate itself is not affected by moisture condition.

Similarly, Aggregate 6 shows no evidence of effect of moisture content on durability in the air-entrained mixes. In the 3000 psi non-air-entrained mixes, durability factors were 15 and 5 for the dry and soaked aggregate, respectively, with corresponding air contents of 3.0 and 2.3 percent. Did the aggregate become less durable with soaking or is the lower durability caused indirectly by the lower air content available to protect the paste? In the 4500-psi mixes, air contents of 2.6 and 2.7 are listed for the dry and soaked aggregate, respectively, with corresponding durability factors of 28 and 12. Considering the general accuracy and reproducibility of air content and freezing and thawing tests and the possibilities of different air bubble characteristics in the two mixes, it does not seem safe to assume, even in this case, that soaking the aggregate was the direct cause of lowered durability.

The procedure used in obtaining the durability factor of 12 for the soaked aggregate mix is not clear since the test was apparently discontinued at 75 cycles with the specimens still having 76 percent of their original E value (neither the failure point of 60 percent E nor the end point of the test, 300 cycles, had been reached). Calculation of the durability factor under such conditions necessarily involves some assumption as to the rate of deterioration that would be produced by continuation of the test. Unless the wet and dry aggregate mixes are both kept in test until the selected failure point is reached or for the same number of cycles, an accurate comparison of durability is difficult.

If aggregates are to be evaluated by freezing and thawing tests on concrete as suggested in the paper, either to determine relative durability of different aggregates or the effects of moisture condition on a given aggregate, it seems essential that (1) the cement paste must be adequately protected to eliminate it as a possible major factor in any deterioration that occurs, and (2) all mixes should be run to a selected level of deterioration or to a specified number of cycles. The first requirement may be achieved by maintaining some minimum air content, kept as constant as possible and by use of freezing in air techniques which minimize the influence of the paste on durability. The second seems essential if valid comparisons are to be made and it would be desirable to base all durability factors on either the number of cycles required to produce a given loss in E or on the relative E value after a specific number of cycles, rather than combining the two as is now done.

These considerations also apply to normal weight aggregates. With the previously mentioned factors of moisture content and curing of the concrete, they will undoubtedly require additional research and investigation before standardization of procedures is possible. The additional question of correlation with field performance must then be considered before specification writing agencies will be in a position to prepare definite and valid requirements. The authors have made a major contribution in this field where so much information is needed. It is important that work of this type continue so that improvements may be made in concrete durability requirements.

AUTHORS' CLOSURE

The authors are most appreciative of the comments and criticisms offered by the discussers.

Mr. Erskine's comment relative to the performance of lightweight aggregate concretes in actual field use provides a valuable supplement to the laboratory findings. Mr. Erskine, in private communication, called the authors' attention to the following errors: In Table 4, in the next to the last column, the absorption should be shown as "percent by volume;" a similar change should be made in the legend for the abscissa of Fig. 5.

Mr. Duey poses several questions which warrant some detail in answering. With respect to the freezing and thawing method used in these tests, the report stated that prior laboratory work indicated that this test procedure produced results comparable to those using ASTM C 290-57T, "Test for Resistance of Concrete Specimens to Rapid Freezing and Thawing in Water (Tentative)." Both of these procedures are much more severe than ASTM C 291, which involves freezing in air. A good correlation may or may not exist between the water-freeze and air-freeze methods, particularly when comparing the results of different laboratories. For more detailed information on comparison of freezing

and thawing methods, the readers are referred to Highway Research Board Special Report 47.*

In response to Mr. Duey's question concerning the grading of the aggregates, the grading was kept the same as that used for the same aggregates in the work by Shideler.† Insofar as the durability of these concretes is concerned, the differences in grading between that selected and as received would not be expected to alter the conclusions reached.

Mr. Duey wonders whether an explanation is available for the vast improvement noted in the performance of air-entrained concrete made with Aggregate 6 as compared with the non-air-entrained concrete made with this aggregate. The authors have no explanation for this specific occurrence, except to note again that the use of entrained air generally resulted in substantial increases in durability. Like Aggregate 6, other aggregates showed changes in relative standing when comparing the relative standings in non-air-entrained with those in air-entrained concretes. For example, in the 3000-psi concretes, in 8 out of 20 cases the relative standing of an aggregate was improved by air entrainment; while in the 4500-psi concretes, this occurred in 10 out of 20 cases. The jumps were not as dramatic as for Aggregate 6, nevertheless they did occur. The authors believe it would be unwise to assign relative standings for the aggregates based on these tests, but if the temptation is difficult to resist only the performance in air-entrained concretes should be considered.

With respect to Mr. Duey's comments as to the performance of certain of the lightweight aggregates relative to the Elgin sand and gravel, the authors pointed out that only *one* normal weight aggregate having a good service record was selected and that such data should be expected.

Mr. Duey's closing remarks concerning moisture content are pertinent. The effect of moisture content of aggregate was, however, of much greater consequence for the non-air-entrained concretes and this was so stated in Comment 3. Despite the use of aggregates dried to only 0.1 to 0.2 percent moisture, the durability of many of the non-air-entrained concretes was quite low. Soaking for 1 hr would have resulted in moisture contents of lightweight aggregates ranging from 2.2 percent for one of the fine aggregates to as high as 11.8 percent for one of the coarse aggregates, as can be seen in Table 2. Even this relatively short period of soaking would have a significant influence on the durabilities of the non-air-entrained concretes, and for certain of the aggregates in the leaner air-entrained concretes. The authors suggested that the capabilities of a particular aggregate be evaluated by laboratory tests. Such tests could include moisture content as a variable, providing the pro-

* "Report on "Cooperative Freezing-and-Thawing Tests of Concrete," *Special Report* 47, Highway Research Board, 1959.

†Shideler, J. J., "Lightweight—Aggregate Concrete for Structural Use," ACI JOURNAL, *Proceedings* V. 54, No. 4, Oct. 1957, pp. 299-328.

ducer and user with valuable information with respect to the influence of this feature of aggregate usage.

DURABILITY

Mr. Lewis' lengthy discussion supplies much supplemental information with respect to evaluating concrete durability. Mr. Lewis suggests that the 14-day drying at 50 percent relative humidity during the curing prior to the start of tests is not ordinarily included in tests of normal weight concrete. This is true when referring to the four tentative ASTM methods of conducting freezing and thawing tests. The authors felt that almost all concretes undergo drying before being subjected to freezing and thawing and that therefore a prior curing period which included an air-drying period resembled actual exposure conditions in a more realistic manner than did continuous soaking. Air-drying generally results in a material increase in durability, since on resaturation only a part of the water lost is reabsorbed, resulting in a lower moisture content and therefore less potential for distress on freezing. In the series of tests cited by Mr. Lewis, significant drying, although unplanned, undoubtedly took place during the air-freeze portion of the cycle in the time it takes to produce freezing in the surface region. Even after freezing, ice or moisture loss can take place depending on conditions within the freezing chamber.

Attempts to define precisely the independent roles of variables such as moisture content and other properties of aggregates can be frustrating, as Mr. Lewis' comments would indicate. These tests are not sufficiently detailed to provide such definitive information for each aggregate in this study, particularly when considering the data for the non-air-entrained concretes. Mr. Lewis pointed out that in the majority of cases the non-air-entrained concretes made with the air-dried aggregates generally had higher air contents than the non-air-entrained concretes made with the soaked aggregates. However, these air content differences do not correlate well with the increases in durability noted for these concretes. For example, in the 3000-psi concrete made with Aggregate 3, the additional 0.2 percent air was accompanied by an increase in durability factor from 59 to 82; for Aggregate 5, however, a *decrease* in air content of 0.3 percent resulted in an increase in durability factor from 29 to 89 when changing from soaked to air-dried aggregates. Even for the 3000 psi normal weight concrete, a decrease of 0.2 percent air resulted in an increase in durability factor from 28 to 60 when changing from soaked to air-dried aggregate. Other instances of poor correlation are evident. It would be expected that these additional increments of air might be of more benefit in those cases of extremely low durability of non-air-entrained concretes made with soaked aggregates, yet such was not the case.

The authors must take exception to Mr. Lewis' statement that the nature of the air (whether entrapped or entrained) and its effect on durability of the paste is unknown. It is well established that the intentionally entrained air provided by acceptable air-entraining admixtures or air-entraining additions to hydraulic cements produces an air-void system that is highly efficient in protecting the paste against the effects of freezing and thawing. In the absence of these materials, variations in the content of what Mr. Lewis calls "entrapped" air result in only low-order influences on durability.

Mr. Lewis called attention to the calculation of the durability factor for Aggregate 6 in the 4500 psi non-air-entrained concrete made with the soaked aggregate. The durability factor as shown is correct. The authors regret, however, that the expansion, percent of original *E*, and weight change data shown are for 50 cycles, rather than 75 as indicated. These data should be:

Expansion, percent	0.35 (75)
Percent of original *E*	30 (75)
Change in weight, percent	+0.6 (75)

Interpolation between 50 and 75 cycle values for percent of original *E* values of 76 and 30, respectively, yields a value of 60 percent of original *E* at 59 cycles, resulting in a durability factor of 12 when using 300 cycles as the end point of these tests.

The authors again wish to express their appreciation to the discussers of this paper.

Discussion of a paper by Frank E. Legg, Jr. and William W. McLaughlin:

Gravel Beneficiation in Michigan*

By DELMAR L. BLOEM, WALTER H. PRICE and GEORGE B. WALLACE, and AUTHORS

By DELMAR L. BLOEM†

The authors have presented worthwhile information on a subject of great economic importance, viz., that of utilizing aggregates from substandard deposits under the exacting requirements of severe exposure to weather. Economy, as might be expected, is a key consideration. Beneficiation will upgrade some aggregates to acceptable quality at reasonable cost, but not others. Whether or not an aggregate can be economically treated, and what type of treatment should be considered, will depend on the nature of the parent material and of the deleterious constituents. Further, the processing cost which can be justified will depend on the use to which the concrete will be put.

The Joint Research Laboratory of the National Sand and Gravel Association and National Ready Mixed Concrete Association has conducted tests to evaluate the effectiveness of aggregate beneficiation in improving the resistance of concrete to freezing and thawing. Data on heavy media treatment were published earlier.‡ Previously unpublished results are summarized briefly in Table A.

In general, comparisons were made between unprocessed aggregate and the same aggregate after treatment. In some cases, different degrees of processing were employed and, in others, the comparisons included evaluation of the deleterious portion of the aggregate. Routine acceptance tests were made on the aggregates except in a few cases where quantities were too limited.

Standard testing procedures of the ASTM were employed wherever applicable. Tabulated values of "total deleterious particles" include all pieces judged to be potentially unsound when exposed to freezing and thawing in concrete. These included shale, porous and weathered rocks, lightweight chert and soft particles. The freezing and thawing tests

*ACI JOURNAL, *Proceedings* V. 57, No. 7, Jan. 1961, p. 813.
†Member American Concrete Institute, Associate Director of Engineering, National Sand and Gravel Association and National Ready Mixed Concrete Association, Washington, D. C.
‡ "Heavy Media Treatment of Gravel," *Circular* No. 55, National Sand and Gravel Association, Washington, D. C.

TABLE A — EFFECTS OF BENEFICIATION TREATMENTS ON GRAVEL

Gravel source No.	Reference No.	Description of processing	Percent deleterious particles		Absorption, percent	Sodium sulfate loss, percent	Los Angeles abrasion loss, percent	Freezing and thawing durability factor, 100 cycles
			soft	total				
	50	Untreated	0.9	13.5	1.2	5.0	21.0	24
1	49	Product of elastic fractionation	0.4	7.7	1.0	2.7	19.4	27
	51	Discard of elastic fractionation	15.4	29.2	2.7	20.0	33.7	23
	54	Untreated	16.3	40.1	2.6	13.2	30.4	4
2	55	Product of elastic fractionation	1.2	15.9	1.4	4.6	26.3	26
	53	Discard of elastic fractionation	22.4	51.2	4.1	24.1	34.6	5
	59	Untreated	2.1	15.7	1.3	4.8	21.3	36
3	58	Product of elastic fractionation	0.1	11.4	0.9	2.6	17.8	57
	60	Discard of elastic fractionation	5.8	20.6	2.1	8.4	27.1	12
	87	Untreated	3.2	12.7	1.4	6.3	19.8	34
4	91	Product of elastic fractionation	0.0	7.9	1.2	3.0	16.6	38
	89	Untreated	4.1	14.7	1.4	5.2	18.7	10
5	88	Product of elastic fractionation	1.2	7.9	1.1	2.8	17.7	40
	92	Product of elastic fractionation with light chert removed	0.9	4.8	1.1	4.2	18.6	68
	47	Untreated	0.0	18.1	2.5	17.2	29.7	25
6	46	Treated in impact crusher	0.8	12.8	2.4	17.6	28.8	70
	2783A	Aggregate 50 percent lighter than 2.55 specific gravity	—	—	2.2	—	—	3
7	2783B	All aggregate heavier than 2.55 specific gravity	—	—	0.4	—	—	50
	3251	Untreated	—	—	1.0	—	—	61
8	3252	Product heavy media (sink in 2.51 specific gravity)	—	—	0.7	—	—	76
	3253	Product heavy media (sink in 2.57 specific gravity)	—	—	0.6	—	—	90
	3290	Untreated	—	—	3.2	—	—	65
9	3291	Product heavy media (sink in 2.50 specific gravity)	—	—	2.3	—	—	83
	3292	Waste from heavy media (float in 2.50 specific gravity)	—	—	6.6	—	—	54

were made on 3 x 4 x 16 in. concrete beams in accordance with ASTM C 291, "Tentative Method of Test for Resistance of Concrete Specimens to Rapid Freezing in Air and Thawing in Water."

In Table A, data for the first five sources compare unprocessed aggregates with the same materials treated by elastic fractionation. In all cases, treatment resulted in improved performance as measured by routine aggregate acceptance tests. Deleterious particles, absorption, sulfate soundness loss and abrasion loss were reduced in varying degree. The processed aggregates also showed improved resistance to freezing and thawing in concrete, but the degree of benefit differed greatly. The increase in resistance ranged from negligible for Source 1 to considerable for Sources 2 and 5.

For Sources 1, 2, and 3, tests were also made of the discard portion from elastic fractionation. As would be expected, this material in each case performed less well than the unprocessed parent aggregate, verifying that the treatment was removing the less desirable constituents. An extra condition was studied for Source 5; the product of elastic fractionation was treated further by removing lightweight chert manually. This provided additional improvement in the resistance of concrete to freezing and thawing, but had little effect on sulfate loss, absorption or abrasion loss.

Data for Source 6 show some mild contradiction for comparisons between untreated aggregate and aggregate passed through an impact crusher to reduce friable particles. The treatment reduced deleterious particle content from about 18 to 13 percent but did not reduce absorption, sulfate soundness loss or abrasion loss. There was a marked increase in resistance to freezing and thawing of the concrete. It appears from these results, as well as those for Sources 1 through 5, that the routine acceptance tests are not always a good indication of the potentialities of beneficiation treatments.

Evaluations of heavy media treatment were made for Sources 7, 8, and 9. Source 7 involved comparisons between two synthetically compounded aggregates containing a suspect lightweight material. A separation of the questionable material was made between fractions lighter and heavier than 2.55 specific gravity. The heavy and light fractions were used to constitute 50 percent of the total coarse aggregate, the remainder being a sound quartz gravel. The heavier fraction showed much the better performance in the freezing and thawing tests of concrete.

In the case of Source 8, untreated gravel was compared with the sink portion from heavy media treatment at specific gravities of 2.51 and 2.57. Freezing and thawing resistance was improved by the processing and was better for the higher of the two operating specific gravities. Gravel from Source 9 was compared with the sink and float fractions

secured from it in heavy media treatment at a specific gravity of 2.50. As expected, best performance was attained from the sink portion, poorest from the float portion (which would normally be discarded) and intermediate from the unprocessed aggregate.

These tests confirm that the feasibility of processing to upgrade aggregates must be carefully evaluated for the particular source. Depending on the nature of the deposit, a method may be found to improve the chances of meeting specification requirements. It may not always follow, however, that the effects of processing, or lack thereof, on acceptance test results will be reflected in the performance of concrete.

By WALTER H. PRICE and GEORGE B. WALLACE*

Producer efforts to meet the expanded demand for high-quality aggregates in southern Michigan have made this area a foremost "proving ground" for gravel beneficiation techniques. The authors' recapitulation of experiences with several beneficiation methods should prove particularly valuable to those concerned with selection of a process for upgrading gravel deposits contaminated by deleterious particles. The admonition to tailor the process to the requirements of each deposit is borne out by experience with various types of materials, but usually the heavy media separation process gives best performance.

In light of the authors' conclusions regarding the value of heavy media separation, Bureau of Reclamation experiences with this process may be of interest. During the investigative phases for Glen Canyon and Flaming Gorge dams, it was found that aggregates from those sources which were economically available for these projects contained appreciable amounts of materials of poor physical quality. Most of the offending materials were calcareous and shaly rocks. Since these rock types were lighter in weight than the sound constituents, the deposits under consideration seemed particularly amenable to some form of aggregate beneficiation. Elastic fractionation, hydraulic jigging, and heavy media separation methods were investigated in the Reclamation laboratories at Denver. Laboratory trials of the heavy media separation process produced the most promising results in removing lightweight particles from Glen Canyon and Flaming Gorge aggregates. Beneficiation was limited to the No. 8 through 1½-in. fractions because the amount of lightweight material in the plus 1½-in. gravel was not objectionable, and no effective and economical process was available for removing lightweight material from the minus No. 8 sand fractions. A specific gravity of separation of 2.50 was selected for both aggregates to achieve a quality product

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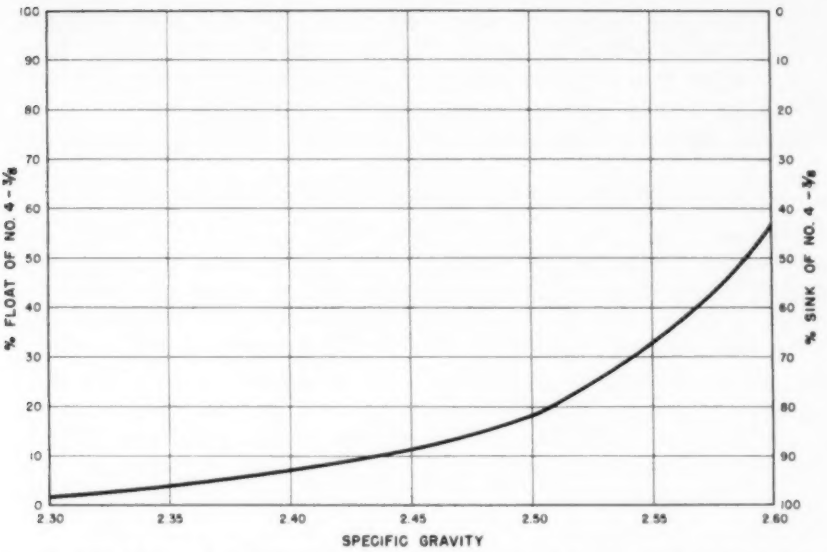

Fig. A—Sink-float specific gravity relationship; about 20 percent of feed has a specific gravity less than 2.50; waste increases rapidly at higher specific gravities

with a minimum amount of waste. Fig. A, which illustrates the results of laboratory "heavy liquid" separation tests of No. 4 to ⅜-in. Flaming Gorge Dam aggregate, gives an indication of the technique used in selecting this separating gravity.

Reclamation specifications permit only 2 percent lightweight materials in the so-called "sink" product from the heavy media separation plant. This limitation has been achieved by intensive study of the mechanics of separation in both laboratory plants and field installations. The critical factors in plant operation, as derived from these experiences, are listed below. These factors influence the effectiveness and economy of separation. They apply to the two distinct types of heavy media separation plants in current Bureau use: the spiral separator and the cone separator. The difference between the two relates only to the separatory vessel. The spiral separator consists of an inclined tank in which a spiral rotates. Aggregates are fed into the separation pool at the lower end of the tank. The heavy aggregate particles sink through the medium and are carried by the rotating spiral to the discharge point at the upper end of the tank. The light particles float on the surface of the separation pool and are carried with the medium overflow through weirs located on the side of the tank opposite that of the aggregate feed. The spiral-type plant is well adapted for rapid processing of coarser aggregate particles.

The cone-type separator makes use of an inverted cone equipped with paddles to agitate the heavy medium in the separation pool. When the aggregate feed is introduced at the pool surface, the heavy particles sink to the bottom of the cone where they are continuously recovered. The float material is discharged over a weir at the top of the cone.

FACTORS INFLUENCING HEAVY MEDIA SEPARATION

It is important to recognize at the outset that not all of the lightweight particles are floated off during the separation process. Some are surrounded and entrapped by heavyweight particles and carried to the bottom of the medium pool and eventually to the batch plant as a part of the sink product. Conversely, not all of the heavyweight particles sink. Some are carried "piggyback" and in suspension with the lightweight particles over the float weirs. If too much of this occurs, the process becomes inefficient and expensive. A high-grade sink product and minimum operating costs are obtained by understanding and controlling the influence of the following five factors in the separation process:

1. Quality and rate of feed
2. Size range of feed aggregates
3. Retention time in separation pool
4. Specific gravity differential of the heavy medium
5. Medium viscosity and slimes in the medium

Operation of the heavy media plant is directly influenced by the quality and rate of feed. A feed that is of high quality, i.e., contains only a small percentage of lightweights, can be processed more rapidly than a low-grade feed. When the quality of the feed changes, the rate of feed must be adjusted accordingly. For a given quality, it is important to maintain a uniform flow of feed into the separator. Surge storage bins located immediately ahead of the heavy media separation plant, with a vibrating feeder to regulate the amount of feed, assist in smoothing out fluctuations occurring within the main screening plant. They also permit continued operation of the heavy media separation plant during brief shutdowns of the main screening and washing plant.

In the Bureau, all attempts to process a composite feed of No. 8 through a 1½-in. range have been unsuccessful. When float weir openings are set to establish a flow deep enough to permit passage of the larger size lightweight particles, many fine, heavy particles are swept by the medium current over the weirs. Also, fine particles are carried on top of the larger lightweight particles. In some field trials with this composite feed, as much as 92 percent of the No. 8 sand was lost with the float product. As a result, the No. 8 sand and the No. 4 to 1½-in. fractions are processed separately at both Glen Canyon and Flaming Gorge dams.

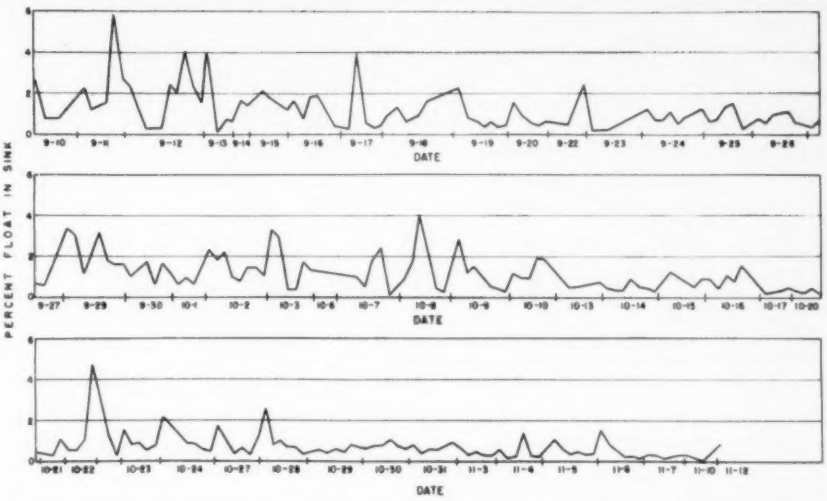

Fig. 8—Heavy liquid test results on No. 8 to No. 4 Glen Canyon sand; specifications prohibit over 2 percent of HMS product to be lighter than 2.50 specific gravity; feed contained about 20 percent lightweights

Retaining the aggregate feed in the separation pool for longer periods permits the agitation within the separator to free entrapped lightweights which would otherwise lower the quality of the sink product. Also, heavyweights riding on top of lightweights have a better chance of being dislodged as the retention time is increased. This is especially important in processing the finer size aggregates. For this reason, a cone-type separatory vessel is used at both projects for processing No. 8 sand. The cone has a greater depth and volume than the spiral separator and is capable of retaining a given rate of feed for a longer period in the separation pool.

The cone separator, in particular, may be operated to obtain a large differential between specific gravities of the medium at the top and bottom of the separating pool. This is helpful in producing efficient separation, especially when there is little difference between specific gravities of many sound and unsound aggregate particles in the feed.

Tests show that the larger the differential, the better the product obtained. The differential in medium specific gravity increases with greater depth of pool, increasing ratios of ferrosilicon to magnetite, and reduced agitation and circulation. These factors also increase the possibility of plugging in the circulatory system, with resultant delays and costly cleanout and restarting expenses. Therefore, it is important to frequently sample the medium near the top and bottom of the pool and

maintain a differential that will provide efficient separation and smooth operation. Although the specific gravity of separation is 2.50 for Glen Canyon and Flaming Gorge dams, separation can be accomplished at any required specific gravity up to about 3.00. When it is desired to separate below a specific gravity of about 2.40, a slurry with magnetite alone may be used; for separation at a specific gravity of 2.80 or above, a slurry of ferrosilicon alone is used.

The specific gravity differential diminishes when the medium becomes contaminated with silt and clays appearing on the surface of the aggregates. These slimes also increase the viscosity of the medium and lower the average specific gravity. It is important, therefore, to continually remove slimes even at the expense of losing a small amount of ferrosilicon and magnetite. This is done by proper regulation of the medium reclamation circuit, which is an essential component of the heavy media separation plant. Proper adjustment will permit removal of most of the slimes without excessive waste of magnetite and ferrosilicon. Average loss of the latter materials at Glen Canyon Dam has been estimated at about 1 lb per ton of sink product.

CONTROL TESTING FOR HEAVY MEDIA SEPARATION

To control the operation of the heavy media separation plant and determine the quality of the sink product, the following tests are regularly performed:

1. Heavy liquid separation
2. Pulp density of heavy media
3. Production measurements
4. Sieve analysis

The Bureau of Reclamation's heavy liquid testing procedure was developed primarily to determine the quality of the sink product produced by the heavy media separation plant. It has also proved valuable in check tests to determine how much sound, heavy material is being wasted with the float rejects and to determine the quality of the feed material. The results of heavy liquid tests indicate the acceptability of the product, the efficiency of the plant, and the quality and uniformity of the feed.

Essentially, the heavy liquid test consists of placing a sample of aggregate from the heavy media separation plant into a liquid of suitable specific gravity, then removing and weighing the lightweight material which floats to the surface. Fig. B shows results of heavy liquid tests at Glen Canyon Dam during production of treated sand. Note from Fig. B that each time the specification limit of 2 percent float in sink was exceeded, it was possible to change the operating conditions and quickly reduce the amount of lightweight material, often to less than 1 percent of the sink product. Although the quality of the product may

be satisfactory, the efficiency of a heavy media plant may be low if large quantities of heavyweight materials are being carried over the float weirs and wasted with the lightweight particles. Increasing the medium density, decreasing the feed rate, increasing retention time in the medium, increasing the specific gravity differential between top and bottom of medium pool will improve the sink product. Unless these factors are coordinated, however, optimum efficiency will not be obtained. By performing occasional heavy liquid tests on the float by-product and feed, the amount of heavyweight material being wasted can be determined and necessary changes made in the operating conditions to improve the plant efficiency.

The control test most frequently made is to determine the density of the heavy medium at various points within the separator. This test is quickly performed by filling a 1-liter beaker with the medium and weighing on a spring scale graduated to show directly the specific gravity of the slurry. Comparison of pulp density tests indicates to the plant operator when it is necessary to add ferrosilicon or magnetite, change the rate of medium circulation, and increase desliming or feed washing efforts.

Feed rate, as well as float and sink product recovery, in tons per hour, are determined for each size of aggregate. To obtain this information, the appropriate conveyor belts are stopped long enough to remove a sample from a 20 to 30 ft length of the belt. The sample is weighed and the speed of the moving, loaded belt is determined with a stopwatch. From these data, the rate of feed, float production, and sink production can readily be computed. The belt samples are quartered and sieved to determine the percentage of each size fraction. The amount of feed, float, or sink product may then be computed for each size fraction.

AUTHORS' CLOSURE

The additional data regarding aggregate beneficiation provided by Messrs. Bloem, Price and Wallace is a valuable supplement to the original presentation. The authors concur with Mr. Bloem with respect to his admonition that the beneficiation process must be economically justified and could add that an enlightened consumer group is likewise necessary. Unless definite benefits can be demonstrated, the added cost of beneficiation will be considered by the untutored as an unnecessary "frill."

The discussion provided by Messrs. Price and Wallace is particularly valuable since it recounts experiences with heavy media separation for two large engineering projects, viz., Glen Canyon and Flaming Gorge dams. Projects of such magnitude receive unusually careful preconstruc-

tion investigation and the solutions to the problems encountered deserve attention by all who may be faced with selecting or processing concrete aggregates where a high degree of frost resistance is mandatory.

Discussion of a paper by Matthys P. Levy:

Precast Grid-Wall for Banque Lambert*

By PHILLIP L. GOULD†

In the analysis of the columns used for Banque Lambert, values are given in Fig. 8 for different assumptions of column restraint. For the assumption of a hinge at the next higher and lower floor, the values seem to be slightly inaccurate. The expression for relative stiffness of the column for this case (Case b) should read

$$K = \frac{\frac{3EI_2}{H}}{\frac{4EI_1}{L} + \frac{6EI_2}{H}} = \frac{1}{\frac{4I_1H}{3I_2L} + 2}$$

instead of

$$K = \frac{1}{\frac{3I_1H}{2I_2L} + 2}$$

as stated for K in Case b.

Since no individual values are given for I_1, I_2, H, and L, the value of the parameter I_1H/I_2L must be evaluated to establish the corrected value for K and percent variation.

From Assumption (a)

$$\frac{1}{\frac{I_1H}{I_2L} + 2} = 0.425, \quad \text{hence } \frac{I_1H}{I_2L} = 0.35$$

Substituting the value for I_1H/I_2L in the corrected expression for K_1

$$K = \frac{1}{\frac{4}{3} \times 0.35 + 2} = 0.405$$

$$\text{Percent variation} = \frac{0.425 - 0.405}{0.425} = -4.7 \text{ percent}$$

*ACI JOURNAL, *Proceedings* V. 57, No. 8, Feb. 1961, p. 865.
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Discussion of a paper by Alan H. Mattock, Ladislav B. Kriz, and Eivind Hognestad:

Rectangular Concrete Stress Distribution in Ultimate Strength Design*

By P. W. ABELES; HOMER M. HADLEY; K. HAJNAL-KÓNYI; J. L. MEEK; LUIS SÁENZ, IGNACIO MARTÍN, and RAFAEL TAMARGO; and AUTHORS

By P. W. ABELES†

In this paper, the authors have given a broad survey of the advantages obtained when the ultimate load design is based on an equivalent rectangular concrete stress distribution. The writer, who has used this method for almost 20 years, intends to limit his contribution to prestressed concrete, as described by the authors in Tables A-4 and A-5. Before, however, doing so, the writer would like to refer to the authors' statement on pp. 886-887 that at failure "the strain in the steel is nearly equal to the strain in the adjacent concrete" with which the writer fully agrees, as it relates to "the normal type of deformed bar," having a distinct yield point. However, as soon as work-hardened deformed bars, high tensile wires or strands are used, there is no distinct yield point and the bond is destroyed solely in the vicinity of fine cracks when high strength concrete and well distributed steel is used. This has the consequence that Assumption 3 is not valid any more. In fact, with small percentages of reinforcement, there will be at failure an apparent steel stress much higher than the proof stress and even in excess of the tensile strength. The writer does not suggest that such an excess stress should be taken into account in the design, but he would like to point out that instead of the yield point or proof stress the tensile strength should be used.

The writer has shown in an investigation of a series of tests on prestressed concrete at the 4th IABSE Congress in Cambridge[29] that the calculated steel stress at failure reached or exceeded the strength of the steel in all cases, in which wires were well bonded (see Fig. A). However, the values on which Tables A-4 and A-5 are based consider lower stresses in the tensile reinforcement at failure than the tensile strength.

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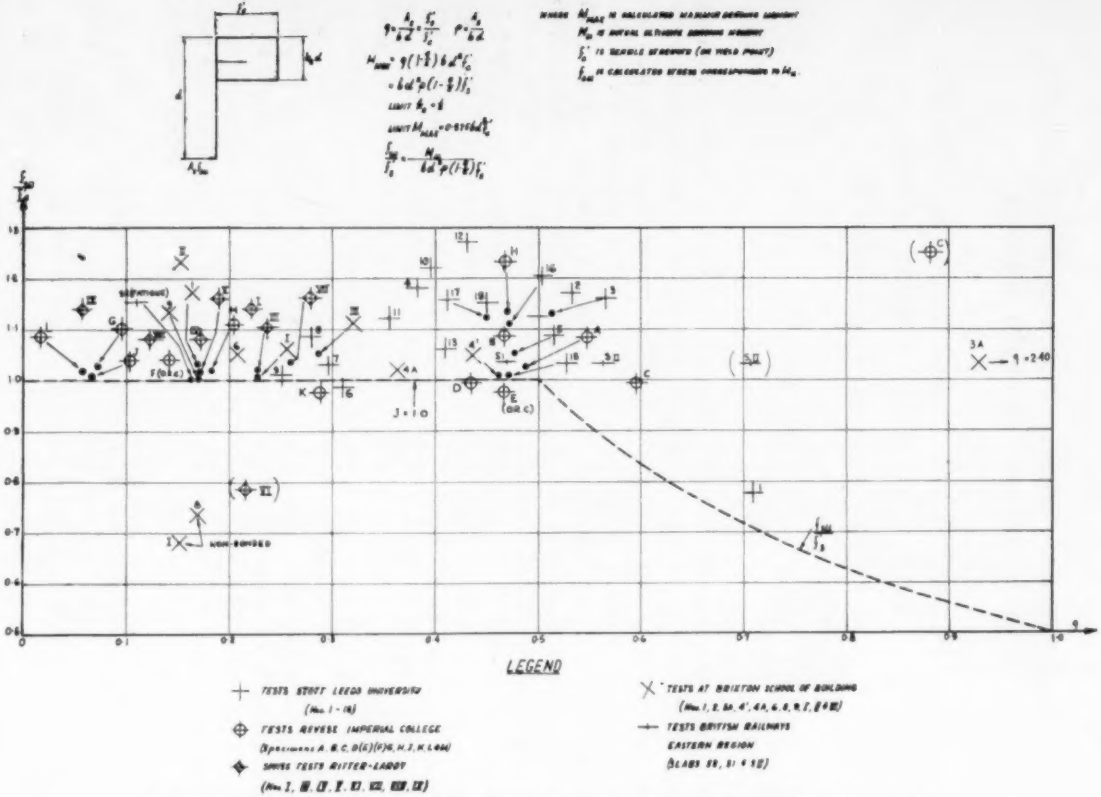

Fig. A—Calculated ultimate steel stress to tensile strength ratio curve

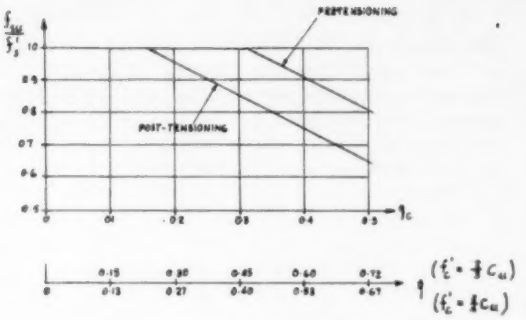

Fig. B—Comparison of pretensioned steel and post-tensioned bonded steel strengths at failure

The method of iteration suggested by the authors based on the shape of the stress-strain curve of the prestressing steel is certainly important for nonbonded steel. However, it appears doubtful whether this kind of calculation is necessary for well-bonded wires close to the tensile edge, where there is no direct relationship between the maximum strain of the steel in a crack and the over-all strain of the concrete at the same layer. Conditions are different with strands and particularly in beams in which the steel is far away from the outer edge of the tensile zone. Consequently, the cracks become much wider than if the steel is close to the tensile edge. This, for example, is the case with the beams described in Reference 23 and discussed in Table A-5, where the strands were 3.7 in. above the outer tensile face.

Generally, there seems to be some discrepancy between the test results in the United States and Great Britain. The high stresses obtained by the writer in his tests with well-bonded pretensioned wires were fully confirmed by Bate's tests of the Building Research Station,[30] who came to the conclusion that in beams with pretensioned steel for a tension reinforcement index $q_c = f_c'q/c_u$ (where c_u is the cube strength and q is the tension reinforcement index pf_y/f_c'), up to 0.3, the entire tensile strength of the steel is reached at failure; while for post-tensioned bonded steel this applies only to $q_c = 0.15$, as is indicated in Fig. B. In Fig. B, in addition to q_c, the index q is plotted for the two limits of $f_c'/c_u = 2/3$ and $f_c'/c_u = 3/4$ (the ratio 0.85 often suggested does not agree with the actual conditions, as the writer has established by many tests). The results shown in Fig. B have been embodied in the British Standard Code of Practice, CP 115: 1959, for prestressed concrete in which the ratios f_{su}/f_s' (i.e., the ratio between calculated ultimate steel stress and tensile strength) is given as 1 for $0 < q_c \leq 0.3$ for pretensioning and $0 < q_c \leq 0.15$ for post-tensioning while for a greater reinforcement index smaller ultimate steel stresses apply as seen in Fig. B.

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By HOMER M. HADLEY*

In reading the introductory paragraphs of this paper, it is noted that its authors contemplated a re-evaluation of the design principles involved, in the light of extensive researches in the United States and abroad that have been made in recent years. Therefore the writer believes that the authors should give close and thoughtful attention to the findings set forth in a paper by Mathey and Watstein.†

The primary finding of that paper was that with steels of equal elastic moduli, the smaller the steel area and the higher the unit stress in the reinforcement the greater become the beams' deflections and the wider the beams' cracks. This is particularly valuable information when high tension steel reinforcement is readily obtainable in the market and the inexperienced may in all innocence be led to use it in shallow, concrete building floors.

But the finding directly pertinent to the authors' re-evaluation is presented in Table 5, p. 1269, of the Mathey-Watstein paper. In this table are set down the observed bending moments at failure of the 12 beams tested, the stresses at failure computed by the straight line theory, and the same stresses computed by ultimate strength theory. They are remarkable values. They should be viewed and considered not only by the authors of this paper but by everyone who has ever advocated the acceptance of ultimate strength design. For what they say is this: If you want to approximate the truth and reality of the natural, physical world, use the straight line theory for the design of reinforced concrete in flexure and forget ultimate design! These are plain, blunt words but the fact is that with the 12 test beams of known size, concrete, reinforcement, loading, and bending moment at failure, this bending moment at failure was approached more closely when calculated by the straight line theory than by ultimate design theory! Such was the case with 11 of the beams; only with the 12th beam was superiority to be assigned to ultimate design! This certainly is an extraordinary discovery.

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†Mathey, Robert G., and Watstein, David, "Effect of Tensile Properties of Reinforcement on the Flexural Characteristics of Beams," ACI JOURNAL, *Proceedings* V. 56, No. 12, June 1960, pp. 1253-1273. Also Discussion, *Proceedings* V. 56, No. 6, Part 2, Dec. 1960, p. 1567.

That a somewhat similar impression was created in the mind of one of the new theory's supporters and is evidenced by the discussion of the Mathey-Watstein paper by Abeles. He writes:

> Table 5 is surprising insofar as there appears to be hardly any substantial difference between the results obtained from the straight line and ultimate strength theories, the former, in fact, giving results which better agree with the observed values. However, this can be explained by the fact that the authors wisely considered only the effect of the steel, since failure was caused primarily by the steel. However, the table might give the impression that the elastic theory gives better results than the ultimate load theory, which would be incorrect. Different results would have been obtained if the maximum bending moments had been computed for the concrete resistance based on the elastic straight line theory with neglected concrete tensile zone.

The writer does not know what may be the practice in England; he does know that in the United States tension in the concrete is always neglected from straight line stress calculations. Therefore this defense of ultimate strength design is possibly slightly irrelevant.

Mathey and Watstein, in their closure quote from Rüsch's paper,* in which he states:

> The agreement between the results of various theories in design, however, is not at all surprising since only the case of under-reinforced beams has often been cited in comparisons with test data. The tensile force in the steel at failure is determined entirely by the yield point; the lever arm of the internal forces is insensitive to assumptions regarding concrete stress. Only tests of over-reinforced members can furnish a true measure of the validity of a flexural theory. There is need for a theory which is not restricted to approximate results in a limited range.

There is one important fact concerning reinforced concrete of which many of the foremost theoreticians seem unaware. In actual commercial construction, the quantity of reinforcement used is a matter of great importance and the result is that only so-called under-reinforced beams are used. How balanced or over-reinforced beams behave is wholly irrelevant to the matter of economical construction.

The problem of design then in the case of the under-reinforced beams is simply this, as Rüsch states:

> The tensile force in the steel at failure is determined entirely by the yield point; the lever arm of the internal forces is insensitive to assumptions regarding concrete stress.

In short

$$(f_{sy})\,(A_s)\ \text{(some certain lever arm)} = M$$

Therefore, this question of the comparative value of two theories of design, confined solely to economical design, resolves itself into the question of comparative lever arms, comparative *jd*'s. Which theory yields a lever arm more closely approximating the reality of the tested beam? In the National Bureau of Standards tests, 11 out of 12 beams gave results favoring the straight line theory; 1 out of 12 favored the

*Rüsch, Hubert, "Researches Toward a General Flexural Theory for Structural Concrete," ACI JOURNAL, *Proceedings* V. 57, No. 1, July 1960, pp. 1-28.

ultimate theory. So what are we to think?

It will be recalled that the tests which largely led to the general acceptance of ultimate strength design in the United States were a series performed in the Portland Cement Association laboratories on U-frames, turned through 90 deg. Those tests involved both bending and direct stress in specimens of awkward shape which, unreinforced as they were in a central zone, required extremely careful handling. So far as the writer knows, no tests similar to them have been made in this country. Why the combination of bending and direct stress should in any way affect the outcome of these tests the writer is unable to see but there is this basic difference between the PCA tests and those of Mathey and Watstein: the one involved bending and direct stress, the other bending only. Since beams designed for flexure only can readily be made and tested anywhere, independent testing can easily be carried out and reported and general agreement can then be reached respecting design theories. It is to be hoped that such testing will be made. Otherwise there is head-on and flagrant contradiction between these two series of tests and proper re-evaluation is necessarily impossible under such conditions. There should also be developed a rational explanation of how concrete, so notoriously weak in tensile strength as to be assumed to have none, can be put under maximum compression and then be assumed to become plastic and possessed of all the tensile strength that that condition requires.

By K. HAJNAL-KÓNYI*

The paper is an excellent summary of the application of ultimate strength design to a great variety of cases. It is interesting to note that the Eq. (1.4), for under-reinforced beams, is identical with the equation proposed by Whitney in 1947.[31] The acceptance of a rectangular stress block for the purpose of design by both the ACI Code and the latest proposition of the European Concrete Committee (CEB) is a welcome step towards the simplification of design without any sacrifice of accuracy.

The writer has been interested for many years in the determination of q_b as a function of both f_c' and f_y.[32] In a recent discussion in the JOURNAL[33] he pointed out that the limit of q_b in the 1956 ACI Code would have to be reduced with increasing f_y and it is gratifying for him that the authors have arrived at the same conclusion in Fig. 10. Their simplified suggestion $q_{lim} = 80/\sqrt{f_y}$ for $f_c' < 4000$ psi is reasonable although the writer prefers to express the limiting value of q as a fraction of q_b, an alternative also mentioned by the authors.

It would appear from Table A-1 that the reduction of q_b as a function of increasing f_c' should be greater than proposed by the authors.

The beams in Table A-1 can be divided into three groups according

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to the value of f_c' and are shown in Table A.

TABLE A — SUMMARY OF ULTIMATE STRENGTH RATIOS (SEE TABLE A-1)

Group	f_c', psi	No. of beams	Average Test/Calculated
1	$2000 > f_c'$	12	1.205
2	$2000 \leq f_c' \leq 4000$	29	1.060
3	$4000 < f_c' \leq 5800$	18	0.943

It may be seen that the calculated values are much on the safe side in Group 1. This has already been demonstrated by Whitney in Fig. 6 of Reference 4. From a practical point of view this group is hardly of any significance. The average of Group 2 shows an excess of 6 percent over unity which is acceptable. On the other hand, the average of Group 3 is on the unsafe side. The maximum ratio in this group, occuring twice, is only 1.02 whereas two values are lower than 0.90, one as low as 0.85. Further, it should be pointed out that Fig. 10 is extended to $f_c' = 7000$ psi although the maximum value of f_c' in the table is only 5800 psi. There are only three beams with $f_c' > 5000$ psi.

It appears that Beam 4407 has been included in this group by error although according to the paper by Lash and Brison[16] failure occurred by crushing of the concrete. However, the q value for this beam is 0.0367 $\times$ 40,800/4170 = 0.359 which is much less than q_b for f_y = 40,000 psi. According to Eq. (1.4A), $M_{ult}/bd^2f_c' = 0.359\ (1 - 0.59 \times 0.359) = 0.283$ and not 0.333 as in the table. Hence the value Test/Calculated = 0.305/0.283 = 1.08 instead of 0.91 in the table. Consequently, if the yield point was, in fact, 40,800 psi, it must have been exceeded at failure and this beam should have been omitted from the table. Even so, the average of the remaining 17 beams is only 0.946.

Group 2 also contains a beam which should not have been included and to which similar considerations apply as to Beam 4407, i.e., Beam 6303. Its q value is 0.0147 $\times$ 72,000/3290 = 0.322 $< q_b$. From Eq. (1.4A) $M_{ult}/bd^2f_c' = 0.322\ (1 - 0.59 \times 0.322) = 0.261$ as against 0.292 in the table. The ratio Test/Calculated = 0.270/0.261 = 1.03 instead of 0.92 in the table. By the omission of this beam the average of Group 2 increases to 1.066. The average of 1.02 of Groups 2 and 3 (combined) conceals the great difference between the individual averages. To obtain a better agreement between test results and calculated values it would be necessary to vary the coefficients in Eq. (1.8) and (1.9) as functions of f_c' to a greater extent than in the paper.

In a recent paper[35] the 59 beams included in Table A-1 have been analyzed in a different way. The analysis is based on the assumption that the ratio M_{ult}/bd^2f_c' decreases linearly with increasing f_c'. The agreement with test results is better than in the present paper but the formula adopted does not take into account the influence of f_y. The advantage of the formula is that in the range $f_c' > 5000$ psi the theoretical values are on the safe side. The ratios Test/Calculated for the three beams in question vary between 1.04 and 1.12 as against 0.92 to 0.95 in the present paper. Beams 4407 and 6303 with ratios of

0.95 and 0.83, respectively, should have, of course, been excluded also from this analysis.

Another possible approach to the problem would be to determine the q_y values based on f_y and to introduce a reduction factor < 1 as a function of q_y/q_b which is to be applied to the hypothetical values of M_{ult}/bd^2f_c' obtained from Eq. (1.4A) by substituting q_y. Alternatively, a factor > 1, also as a function of q_y/q_b, could be applied to M_{ult}/bd^2f_c' corresponding to q_b.

Since the number of beams with $f_c' > 5000$ psi is only 3 and no test data are available for $f_c' > 5800$ psi the writer would like to suggest that more tests should be carried out on over-reinforced beams with f_c' between say 5000 and 8000 psi and with $f_y = 60{,}000$, 75,000, and 90,000 psi, respectively. Lower values of f_y in concrete of this grade are of little practical importance since the percentage would be excessive.

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35. Kriz, L. B., and Lee, S. L., "Ultimate Strength of Over-Reinforced Beams," *Bulletin* D36, Portland Cement Association Development Department.

By JOHN L. MEEK*

The authors are to be complimented on their excellent presentation of the equations for calculating the ultimate strengths of reinforced concrete members in flexure, and combined flexure and axial compression. The paper must surely be one more step toward the general recognition by the civil engineering profession of ultimate strength design of concrete structures as a superior design tool to the classical method now in use.

Two points come to mind, which although not critical to the progress of ultimate strength design, perhaps should be considered.

The first is the question of combined bending and axial tension. Such a case occurs frequently in beam-column construction, when shrinkage and temperature forces occur, and must also occur in shell

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structures. If one extrapolates the *P-M* interaction curve for a member subjected to combined bending and axial compression into the tension zone, it would appear that small tensions can produce large reductions in the ultimate moment (see Fig. C).

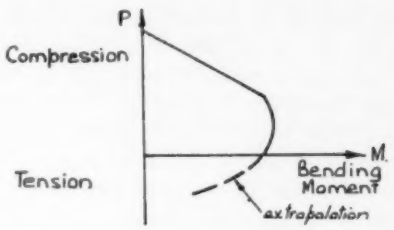

Fig. C—*P-M* interaction curve of reinforced concrete member

The writer would like to know if any experimental work has been done in this region and if any recommendations for design are forthcoming.

The second point of some importance is in the calculation of the ultimate strengths of beams reinforced with high tensile steel (deformed and square twist). Such steel does not have a well defined yield point and its stress-strain curve is similar to Fig. D.

For the beams tested in flexure by the writer, where the steel percentage was less than the balanced percentage (as calculated from Eq. (1.10), the calculated ultimate strengths using Eq. (1.4) (modified for the short term loading by using f_c' instead of $k_3 f_c'$), and a steel stress as 60 kips per sq in., were far below the observed strengths. Much better agreement was obtained if the ultimate strength was calculated, assuming $\varepsilon_u = 0.003$ and using an approximate neutral axis to obtain the corresponding steel strain, and hence from Fig. D, the steel stress. In the beams considered this gave the steel stress as high as 66 kips per sq in., a 10 percent increase.

In such cases the writer feels that the use of the nominal yield point should be replaced by an estimate of steel stress obtained in this manner (using, if warranted, a more conservative ε_u, say, 0.002).

Finally, the writer would like to emphasize the need for further research with reinforced concrete beams at service load.

Service load conditions must never be overlooked by designers using ultimate strength methods. Although current PCA research has been directed toward formulating the Building Code, it must eventually be applied to all fields of reinforced concrete design. Many structures will obviously have more severe exposure conditions than those encompassed by the Building Code. In such cases it will be

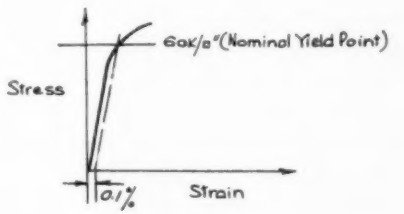

Fig. D—Stress-strain curve of high tensile steel

imperative to know the extent of cracking at service loads and what upper limit on crack width can be tolerated. For example in water retaining structures, no cracking can be tolerated. Thus research should also be directed towards a method for calculating a reliable estimate of crack width. Such work could well be the death blow to plain reinforced bars (in countries where they are still in popular use) and also to the prohibition of the use of high tensile reinforcing in bridge structures where the corrosion hazard is high.

By LUIS SÁENZ, IGNACIO MARTÍN, and RAFAEL TAMARGO*

The paper presented by Messrs. Mattock, Kriz, and Hognestad reveals the extensive work accomplished by the authors in selecting, analyzing and verifying innumerable experimental results. Their aim is well oriented towards a goal of undisputable value to all structural engineers: the establishment of practical hypotheses for the behavior of concrete, which will permit the prediction of the ultimate strength of practically all reinforced concrete members subjected to all types of stresses using simple formulas.

Keeping the main purpose in mind, the average results obtained are satisfactory. However, to calculate the ratio Test ultimate strength/ Calculated ultimate strength without establishing classifications for the 334 cases analyzed is to lose the opportunity of distinguishing clearly which type of members and stresses produce the most approximate results between the calculated values and the values obtained from the tests. More specifically, in the case of beams subjected to flexure, the type of test best suited to verify the stress distribution hypotheses in the compression zone, is the test for concrete rectangular beams reinforced in tension only with the ultimate strength controlled by compression (Table A-1). If an attempt is made to compare the approximation of the theory with reality, it does not seem opportune to mix the results of these tests with those of other tests, such as those for beams reinforced in compression, prestressed beams, or beams with ultimate strength controlled by tension, in which the approximation has much less influence over the real value obtained by the assumed stress distribution.

Consequently, if we analyze the results of Table A-1 and classify the results according to the strength of the concrete, we obtain the values shown in Table B.

Although for practical purposes the results of this table are acceptable, since the 21 percent difference between the theoretical val-

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ues and the tests take place in concrete below 2000 psi (which is not used often), it is, nevertheless, an index that the hypothesis is solely applicable within acceptable limits for certain qualities of concrete. This has been indicated by the authors in Table A-1, showing average values and the standard deviation excluding those with concrete of less than 2000 psi.

The fact that the margin of applicability comprises the concrete currently used for members subjected to flexure makes the hypothesis useful in practice, but the necessity is felt, from the theoretical point of view, of making a closer approximation to the actual phenomenon for the complete range of qualities of concrete.

Among the possible causes of this divergence between the test results and the calculated values based on the hypotheses presented in the paper, the following objections may be noted.

GENERAL ASSUMPTIONS

Of the five general assumptions indicated on p. 876, objections may be raised to Assumptions 1 and 4.

As to the first, it could be said that it does not seem logical to assume a value for the constant k_1 between 0 and 4000 psi and from this point on to diminish its value proportionally with increments in the quality of the concrete. From Fig. 4, it may be inferred that the value of k_1 could be portrayed more easily and aptly with a single continuous linear function than with two straight lines, as there is no apparent reason for

TABLE B — SUMMARY OF ULTIMATE STRENGTH RATIOS

Strength of concrete, psi	Test/Calculated (average)	No. of tests
1000-2000	1.21	12
2000-3000	1.06	12
3000-4000	1.06	17
4000-5000	0.94	14
5000-6000	0.95	4

TABLE C — COMPARISON OF EXPERIMENTAL AND CALCULATED ULTIMATE STRENGTH VALUES

f_c', psi	$f_u = 5 f_c'^{0.8}$, psi	$f_u = 0.85 f_c'$, psi
0	0	0
1000	1250	850
2000	2180	1700
3000	3030	2550
4000	3800	3400
5000	4550	4250
6000	5130	5000
8000	6630	6800
10000	7930	8500

the horizontal portion indicated. Fig. 12 of Reference 10, shows a single linear expression for k_1, which seems more acceptable. This same figure indicates that k_3 is not a constant, and even in the most horizontal portion of the curve its average value is higher than 0.85.

As to the design maximum strain in flexure (Assumption 4), assumed as 0.003, it seems to be an extremely conservative value from one point of view, and still it does not represent the real tendency of the phenomenon. In fact, Fig. 6a shows that the ultimate strain in flexure could be represented by a line with a well defined slope which shows that the strains decrease from the 1000 or 2000 psi towards the higher values of 10,000 psi and over. This means that the phenomenon is not correctly portrayed by a horizontal line. Besides, this figure shows that the value of 0.003 is below almost all the experimental results shown. It is true that this criterion is conservative and it may be considered as an additional safety factor, but it does not correspond to the purpose of the paper which is to determine the most exact values possible for the ultimate strength.

VALUES OF f_u

Another aspect that influences the theoretical results of the flexural strength is the value adopted for the extreme fiber stress in compression (f_u). The authors state on p. 883 that in accordance with a PCA investigation it has been found that the relation between f_u and f_c' can be expressed closely as $f_u = 5f_c'^{0.8}$ which serves the authors as a basis for establishing an even simpler expression which for all practical purposes is $f_u = 0.85f_c'$.

However, neither one of these two expressions seem to be adjusted to the experimental results. If we assume values of f_c' between 0 and 10,000 psi, we arrive at the values of f_u shown in Table C.

It can be appreciated at first glance that the expression $f_u = 5f_c'^{0.8}$ does not give correct results for concrete qualities of 0 to 3000 psi, since the values of f_u are greater than f_c' which is not possible. For strengths greater than 3000 psi, the value of f_u deviates from f_c' directly with increases in the strength of the concrete. The expression $f_u = 0.85f_c'$ gives values which approximate those given by equation $f_u = 5f_c'^{0.8}$ maintaining a conservative margin with respect to the latter up to 6000 psi.

As a check Fig. E indicates the values of f_c' and f_u obtained from the experimental values shown in Fig. 9 of Reference 10. This graph shows the curves corresponding to equations $f_u = 5f_c'^{0.8}$ and $f_u = 0.85f_c'$ as well as a curve representing the average experimental values.

It can be observed that the differences between the proposed formulas and the experimental one reach quite appreciable amounts for concrete between 2000 and 6000 psi strength, which makes it reasonable to sup-

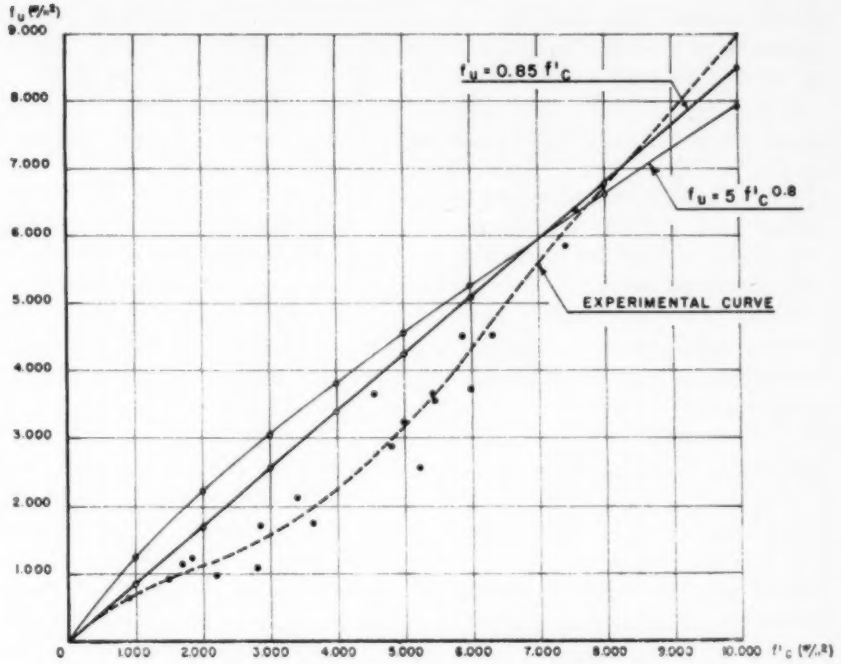

Fig. E — f_u versus f_c relationships

pose that an expression which more faithfully represents the experimental values should be sought.

RESULTS WITH LIGHTWEIGHT CONCRETE

Fig. 4 and 6b show graphs corresponding to the values of k_1, k_2, k_3, and the ultimate strain in flexure. They also include the results obtained from the tests performed by PCA with lightweight concrete.

The graphs of Fig. 4 indicate that the values for lightweight concrete, although close to the values for standard concrete, are systematically lower than the latter. Fig. 6b indicates that the ultimate strains of standard concrete show a tendency to decrease with an increase in the strength of the concrete, whereas the points corresponding to lightweight concrete show a tendency in the opposite direction. This permits the assumption that sufficient coincidence in the values of both types of concrete does not exist to apply the criterion that the strains and factors k_1, k_2, and k_3 of one type of concrete are similar to the other. It is proper to consider the convenience of obtaining additional data for lightweight concrete from 5000 to 10,000 psi. To give equal weight to the results

obtained with standard and lightweight concretes, with the general information available may give rise to final conclusions which are not as correct as they should be.

If standard concrete is considered as a structure consisting of coarse aggregate maintained in position and converted into a resistant element by the action of cement paste, sand, and water which holds it together, it would have to be demonstrated that the coarse aggregate used in the preparation of lightweight concrete has approximately the same mechanical properties as the normal stone used in standard concrete; then, if this is proved to be so, the results of the two types of concrete can be used to form the average value.

As a summary of the foregoing, it may be noted that in Eq. (1.9) for the ultimate moment resistance for rectangular beams controlled by compression.

$$M_{u1} = (0.85\, k_1 f_c')\, bd^2\, k_u\, (1 - k_3 k_u)$$

the values assumed for k_1, k_2, k_3, and k_u have a definite influence. As previously expressed, the actual values of k_1, should be higher than those adopted in the article for a concrete strength less than 4000 psi, being higher for a decrease in strength of concrete. The value of k_3 should also be higher than 0.85, especially for the lower quality concretes. In addition, the value of k_u, which depends on the ultimate strain in compression, should be much higher for the low strength concretes. This explains why the experimental results give values for the ultimate moment of resistance which are appreciably higher than the calculated values, for the lower quality concretes, becoming more correct for the higher strength concretes.

Another aspect of the paper, which we think it is opportune to comment on, is Eq. (7.1) for the design of concentrically loaded columns.

$$P_u = 0.85 f_c' A_c + f_y A_{st} \qquad (7.1)$$

The factor that affects the expression $f_c' A_c$ of the first term of this equation is a function of the concrete strength and of the slenderness ratio of the member. On the one hand, the ratio of the strength of a concrete prism to the test piece taken as reference (a cube or a cylinder) is a function of the strength of the concrete. This aspect has been observed in German practice (DIN-1045) which tabulates the ratios between the prismatic strength and the corresponding cube strength with a series of values from 0.9 for concrete of approximately 1700 psi up to 0.60 for 8500-psi concrete. On the other hand, regarding the variation of strength with the slenderness ratio, we can refer to the experiments of Bach, Gonnerman, and others, who found that when compared with cube strength, prism strength decreases with the slenderness ratio, having values from 1.38 for a slenderness ratio of 0.5 down to a prism strength of 0.84 for a slenderness ratio of 12, according to Bach.

From this discussion it appears that the value of 0.85 chosen by the authors corresponds to the average of a certain group of concrete strengths and a definite range of low slenderness ratios. Therefore, it would prove interesting to have an additional statement in connection with the expression offered which would: (1) define the limits of application of the formula and (2) provide an expression for modifying the factor when it falls outside of the limitation of the formula itself.

Naturally, the ideas expressed herein are applicable not only to concentrically loaded columns, but to all other cases of members in compression as well.

It can be presumed that it is possible to make adjustments in the theory presented by the authors which would permit: (1) the further simplification of the theory, (2) a better adaptation to the experimental results, (3) the elimination of lack of precision in the results, and (4) an increase in the field of application to include concrete of all strengths; even though different coefficients have to be established for standard and for lightweight concretes.

AUTHORS' CLOSURE

The discussions of Messrs. Hajnal-Kónyi and Sáenz, Martín, and Tamargo point out that the stress block coefficients proposed by the authors underestimate the strength of over-reinforced beams with low strength concrete, and overestimate the same in the case of beams with high strength concrete. Although this observation is correct, the authors do not feel that a readjustment of the stress block coefficients is warranted. From a practical point of view, the over-reinforced beams are of less importance than the other types of members for which a comparison between theoretical and experimental strengths was made. By grouping all the specimens considered according to their concrete strengths, the results given in Table D were obtained.

Beams 4407 and 6303 of Table 1 were omitted from this analysis. When all the specimens in any given concrete strength group are considered, the proposed method appears to be conservative in all but the $7000 < f_c' \leq 8000$-psi group, which comprised only four specimens.

The general assumptions were critically examined by Messrs. Sáenz, Martín, and Tamargo. Although the variations of k_1k_3 and of k_2 could be expressed by single continuous functions of f_c', the advantage of dealing with constant design values for the common range of concrete strengths is obvious. The comparison of the factor k_3 suggested in Reference 10 with that proposed for the rectangular stress block is not relevant. In the first case, the factor k_3 relates the cylinder strength to the maximum flexural stress, while in the second to the stress in the equivalent stress block.

TABLE D — VARIATION OF TEST STRENGTH/CALCULATED STRENGTH WITH CONCRETE STRENGTH

Item	Concrete strength f_c', psi							
	1000 to 2000	2001 to 3000	3001 to 4000	4001 to 5000	5001 to 6000	6001 to 7000	7001 to 8000	8001 to 9000
No. of Specimens	49	58	63	78	62	14	4	2
Average, P_{test}/P_{calc}	1.08	1.06	1.07	1.00	1.00	1.04	0.96	1.04
Standard deviation	0.114	0.112	0.087	0.068	0.087	0.069	0.177	0.022

The proposed value of ultimate strain $\varepsilon_u = 0.003$ should not be considered as overconservative. In design, the term ultimate strain means the maximum strain attained when the maximum load is reached. The strain can increase considerably beyond this value without any increase, or even with a decrease, in the applied loads.

EXTREME FIBER STRESS AT ULTIMATE STRENGTH

The discussion of the value of the extreme fiber stress at ultimate strength, f_u, appears to be based on a misunderstanding of the symbol. The experimental values of f_u shown in Fig. E are not in fact values of f_u if, as in the paper, f_u is defined as the stress at ultimate strength in the extreme fibers of a rectangular under-reinforced concrete beam. The points plotted in Fig. E represent the stresses corresponding to the maximum strains attained in the test specimens reported in Reference 10. The stress f_u can be obtained from the stress-strain curves only by a process of numerical or graphical integration. The values of f_u obtained in this manner are plotted in Fig. F.* The agreement between these values of f_u and those obtained by use of the expression $f_u = 5f_c'^{0.8}$ is evident in this figure. Furthermore, the stress-strain curves for low strength concretes show that the maximum flexural stress is higher than the cylinder strength. This excess of flexural over cylinder strength explains why f_u can be greater than f_c' for low strength concretes.

LIGHTWEIGHT CONCRETE

Messrs. Sáenz, Martín, and Tamargo take issue with the use of the same values of coefficients k_1, k_2, and k_3, and ultimate flexural strain, ε_u, for lightweight as for normal weight concrete. It does appear in Fig. 4 that the values of k_1k_3 and k_2 for the lightweight concrete are systematically slightly less than the values for the normal weight concrete. However, it is considered that the proposed values for design

*This is Fig. 7 of Reference 3.

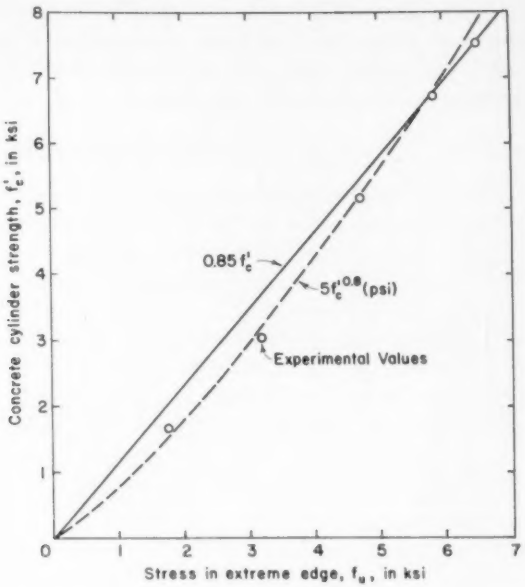

Fig. F — Stress in extreme edge at ultimate strength of reinforced concrete members governed by tension

indicated in Fig. 4 are sufficiently close to the experimental values obtained using lightweight concrete. The proposed ultimate flexural strain of 0.003 is also considered a reasonably conservative estimate for both types of concrete.

COLUMN STRENGTH

Messrs. Sáenz, Martín, and Tamargo question the adequacy of the coefficient 0.85 used in the equation for the strength of an axially loaded column:

$$P_o = 0.85 f_c' A_c + f_y A_s$$

The variation of concrete prism strength with slenderness ratio is acknowledged, but it is considered that this effect would only be apparent in extremely short columns, in which case the coefficient should probably be greater than 0.85. For all other columns, unless buckling is a possibility, the factor 0.85 is reasonable, and is seen to be in close agreement with the factor of 0.84 for a slenderness ratio of 12, which is quoted by the discussers. For slender columns the buckling strength of the columns must of course be used in design.

COMBINED FLEXURE AND AXIAL TENSION

Mr. Meek raises the question of the extension of the interaction diagram for combined axial load and bending into the zone representing axial tension combined with bending. The authors are not aware of

any experimental work concerned with the effect of axial tension on moment capacity. However, it would not appear unreasonable to set up equilibrium and compatibility equations for this case using the assumptions set out in the paper.

HYPER-STRENGTH EFFECTS

The use of compatibility equations to obtain the steel stress at ultimate strength when strain hardening occurs was noted in the original paper. If the steel stress-strain curve beyond yield is known, as in Mr. Meek's case, a closer estimate of ultimate strength can be made by considering compatibility of strains, and hence obtaining the steel stress at ultimate strength. However, in most cases the shape of the stress-strain curve beyond the yield point is not known with certainty. It is, therefore, considered better design practice in these cases to base the calculation of ultimate strength on the specified yield point.

The authors are in complete agreement with Mr. Meek on the necessity for considering service load conditions, and in particular cracking and deflection. An extensive investigation of flexural cracking in reinforced concrete members is at present in progress in the PCA laboratories.

Dr. Abeles questions the applicability to bonded prestressed beams of the assumption that at ultimate strength "the strain in the steel is nearly equal to the strain in the adjacent concrete." It is agreed that in the case of beams prestressed by well bonded thin wires the ultimate strength of the wire may be developed due to high local strains at a crack. However, in the case of beams reinforced with pretensioned tendons of larger diameter this is not always the case. It is therefore considered prudent to calculate the ultimate steel stress in the tendons using the conservative assumption of equality of strain in the steel and the concrete.

STRAIGHT LINE THEORY

Mr. Hadley adduces the results of the tests by Mathey and Watstein of 12 under-reinforced beams as proof of the adequacy of the straight line theory of design. It is common knowledge that for a section reinforced only in tension the straight line theory will yield a value for the internal lever arm which is close to the actual lever arm at ultimate strength. Use of this lever arm together with the yield strength of the reinforcement will therefore result in a close estimate of ultimate strength. This was the procedure followed in the Mathey-Watstein paper. However, this neglects all consideration of the calculated concrete stresses. Calculation of concrete stresses at ultimate strength by the straight line theory would have indicated these stresses as controlling ultimate strength. As pointed out by Mathey and Watstein in the

closure to the discussion of their paper, this would have resulted in the ultimate strength theory yielding a closer estimate of the strength of the beams since it would have predicted tension controlling, while the straight line theory would have wrongly predicted compression controlling.

The purpose of this paper was to develop a theory applicable not only to the case of under-reinforced beams, but also to the case of members subject to all combinations of bending and axial load. The agreement between the predictions of the ultimate strength theory proposed and the measured strength of members subject to many types of loading appears to the authors to justify its use. The effectiveness of the straight line theory in calculating the ultimate strength of much under-reinforced sections cannot counterbalance its shortcomings when applied to the calculation of the strengths of columns and beams with compression reinforcement. These shortcomings have been underlined over the years by the successive empirical adjustments to, and departures from, the true straight line theory in the clauses governing working stress design in the ACI Building Code (ACI 318-56).

Discussion of a paper by W. S. Cottingham, P. G. Fluck, and G. W. Washa:

Creep of Prestressed Beams*

By P. W. ABELES, D. E. BRANSON, W. A. CORLEY and M. A. SOZEN, I. O. OLADAPO and R. D. DAVIES, and AUTHORS

By P. W. ABELES†

In this paper the authors report on the effect of sustained loading over a long period which should be of great importance if all the particulars especially the effective prestress, the resultant stresses under the sustained loading, the development of micro-cracks and visible cracking, were known. Unfortunately, these data have not been given. The reference to "design load" without any qualification often used by research workers is unfortunately of little general value. Of special importance would be to know the magnitude of the tensile stress at the bottom face under the design load. The strains 1 in. above the bottom face clearly indicate that compressive stresses occurred under half the design load while small tensile stresses developed under ¾ the design load and appreciable tensile stresses must have occurred under the full design load.

From the data given it can be assumed that at transfer (i.e., after elastic shortening), the initial tensioning stress was reduced from 140,000 to 130,000 psi and the stresses at transfer due to the prestressing force of $8 \times 0.0352 \times 130{,}000 = 36{,}600$ lb amounted to $2.8 \times 36{,}600/52.5 = 1950$ psi compression at the bottom face and $-0.8 \times 36{,}600/52.5 = -560$ psi tensile stress at the top. In this calculation 52.5 is the cross-sectional area of the concrete and for an eccentricity of the tensioned steel of $e_s = 3$ in. the ratios of the stresses at the outer faces to that at the centroid are $1 \pm 3 \times 3/5 = +2.8$ or -0.8. The stresses due to the self load of 1100 lb are then $\pm 1100 \times 20 \times 1.5/87.5 = \pm 375$ psi (with $Z = 5.25 \times 10^2/6 = 87.5$ in.^{3}). Thus the resultant stresses at transfer are $+1575$ psi at the bottom face and -185 psi at the top face in the middle of the beam.

The concrete stress at the centroid of the steel (Fig. A) is then $1575 - 0.2(1575 + 185) = 1225$ psi. If we assess the maximum losses in the steel assuming shrinkage of, say, 0.03 percent, and creep of $1200 \times 0.3 \times 10^{-6}$,

*ACI JOURNAL, *Proceedings* V. 57, No. 8, Feb. 1961, p. 929.
†Member American Concrete Institute, Consultant, London, England.

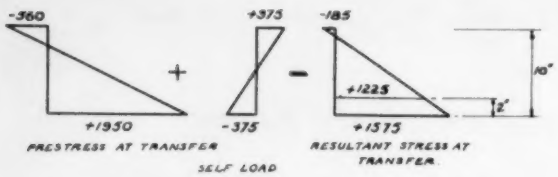

Fig. A—Stresses at transfer, psi

(based on an average concrete stress of 1200 psi and a unit creep of 0.3×10^{-6}), we obtain $(3.0 + 3.6)10^{-4} \times 25.6 \times 10^{6} = 16{,}890$ psi loss in steel stress ($E_s = 25.6 \times 10^{6}$ psi); to this loss relaxation of the steel must be added of, say, 5000 psi, resulting in an effective prestress of $130{,}000 - 22{,}000 = 108{,}000$ psi. Hence the expected minimum effective prestress at the bottom fiber amounts to $108{,}000 \times 1950/130{,}000 = 1620$ psi. Under the design load of 5700 lb we obtain $M = 5700 \times 20 \times 1.5 = 171{,}000$ in.-lb and the stresses are $\pm 171{,}000/87.5 = \pm 1955$ psi. Hence, the stress at the bottom face under the worst conditions may become $1620 - 375 - 1955 = -710$ psi (Fig. B). At such a stress generally, in the writer's experience, cracks should not yet become visible at a static loading, while microcracks would develop at a tensile stress of 450 to 500 psi with a high strength concrete of a 28 day cylinder strength of 6000 psi. The authors' statement that visible cracks were observed at a load of 6000 to 7000 lb (see p. 935) agrees well with this assessment. At these loadings the tensile stresses would be 815 to 1155 psi which is in good agreement with recent tests carried out by the Research Department of British Railways when stresses between 1050 and 1290 psi were obtained, the shortening due to shrinkage and creep in this case being measured and relaxation being assessed.

The authors state on p. 935 that the ". . . results suggested that the beams subjected to full design load had developed tensile cracks during the creep tests . . ." and that "such cracking must have progressed gradually over a period of time . . ." The writer would like to say that this seems to be quite reasonable as far as the matter can be established from the data available. The strains in the tensile zone (Table 4) do not indicate that visible cracks have occurred at first loading and the increase due to creep in 7 years is relatively small. From this it can be concluded that any cracks must have been very fine and well distributed, as it would be expected for such a beam. It may be further considered that when the loading commenced the effective prestress may have been higher than the value computed as the probable lowest effective prestress. Obviously the magnitude of losses depends on the environment, and the entire history of such a test should be known if the tests are to be of general use.

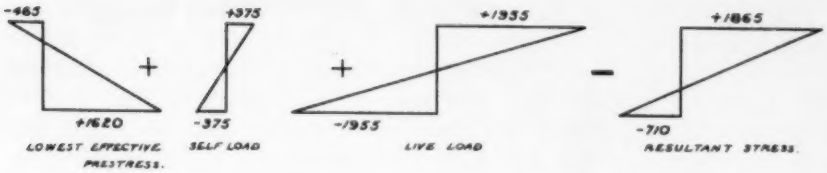

Fig. B—Stresses under working load, psi

British Railways are just commencing sustained loading tests on two specimens whose history is known and the approximate losses of which are known from strain measurements. It would appear that a tensile stress of 850 to 950 psi might be of a critical nature for sustained loading and micro-cracks might become visible in this case. It is intended to make observations to establish whether and when micro-cracks do become visible.

It would be of great importance if the authors were in a position to give further particulars about the entire history of loading (e.g., when and how long was the load of 4000 lb applied) and particularly about the time relationship of temperature and humidity before and during the tests so as to obtain a better basis for assessing the amount of shrinkage and creep. It is well known that creep is greatly affected by humidity and its duration and therefore more knowledge about the variation between 20 and 90 percent humidity during the sustained loading would be desirable.

By D. E. BRANSON*

The authors have reported in an extremely interesting and readable manner experimental results of the time-dependent behavior of prestressed concrete beams under sustained transverse loads in addition to dead load. This work is of interest to the writer in connection with his and Ozell's paper† on time-dependent camber of prestressed concrete beams and is believed to be in an area where experimental information has been lacking. The main differences with regard to time-dependent effects in the prestressed concrete beam, as compared to most of the creep data already available in the literature, is the higher concrete quality, the nonuniform stress distribution in an uncracked concrete element and the variable nature of the sustained prestress force resulting from the loss of prestress. In this connection creep has the dual effect of tending to increase directly camber with time but also tending to decrease camber through the loss of prestress.

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†Branson, D. E., and Ozell, A. M., "Camber in Prestressed Concrete Beams," ACI JOURNAL, *Proceedings* V. 57, No. 12, June 1961, pp. 1549-1574.

For purposes of discussion the terms creep coefficient, camber coefficient, and deflection coefficient will be used in connection with the ratios of time-dependent (total minus initial) to initial creep, camber, and deflection, respectively. The data of the authors and the writer relative to this discussion are shown in Table A. The beam size and span were virtually the same in both cases, but the writer's values, although appearing to reach approximately asymptotic values at the end of 150 days, are obviously somewhat smaller than the corresponding 7-year values would be.

From the data in Table A, the writer would like to make one correlation and perhaps do nothing more than raise a question with regard to his second point. By comparing the deflection and camber coefficients, it is seen that in both cases the coefficients increase with increase of applied stress. Since these coefficients are defined as the ratio of time-dependent to initial deformation, it is indicated by this increase that the time-dependent camber or deflection varies in increasing proportions with stress level, even at the lower levels. If, for instance, twice an initial deformation produces twice a time-dependent deformation (creep proportional to initial strain), the coefficients would be the same.

The creep strains in a nonuniformly stressed concrete member tend to shift the neutral axis, and the time-dependent behavior thus varies with load level and beam fiber considered, particularly when the neutral axis is within the boundary of the beam; i.e., when tensile strains occur. This was clearly reflected in the authors' data for the "½ design load level" in which the gage near the bottom of the beam showed an initial tensile strain but indicated a net compressive creep strain. Since the creep strain distribution is related to the time-dependent deflection or

TABLE A — COMPARISON OF CREEP COEFFICIENTS

Author's data					Writer's data		
		Creep coefficient					
Load level*	Deflection coefficient	Compression (near beam top)	Tension (near beam bottom)	Compression† and tension average	Stress level‡	Camber coefficient	Creep coefficient, average compression
Full design	1.73	2.89	1.29	2.09	4/3 design	1.6	1.9
¾ design	1.45	2.97	0.32	1.65	Full design	1.2	1.6
½ design	1.23	3.19	−0.50	1.35	½ design	0.8	1.4

*Prestressed beams loaded uniformly to full, ¾, and ½ design load for 7 years.
†The writer has taken the liberty of adding this "Compression and tension average" column to the authors' data.
‡Prestressed beams under prestress force and dead load only and prestressed to 25, 45, and 65 percent f_c' (approximately 4/3, full, and ½ design prestress) for 150 days.

camber, perhaps an average of the tensile and compressive creep coefficients of the authors should correspond in a general way to their deflection coefficients. In Table A these average creep coefficients are seen to correspond closely to the creep coefficients obtained by the writer, and in both cases to be a little higher than the deflection and camber coefficients.

It is in connection with this aspect of the time-dependent behavior of prestressed concrete elements that the writer believes more work is needed; that is, with regard to the relation between creep coefficients of the different beam fibers as related to the time-dependent camber or deflection and the corresponding shifting of the neutral axis with time.

By W. G. CORLEY and M. A. SOZEN*

Prestressed concrete became a popular construction medium in the United States at a time when the engineering profession decided to separate the design criteria for safety and serviceability. This applaudable decision has created a vacuum in the routine design procedures. In the design of reinforced concrete, the problem of holding the deflections to serviceable limits had been handled indirectly and sometimes inadequately by the choice of permissible stresses. In prestressed concrete, we do not have the benefit of having used such Machiavellian methods successfully for some 50 years. Yet, to design satisfactory prestressed concrete structures, we should know as much about the deformations of such members as we know about their strength. Consequently, the authors' paper is a valuable and timely contribution.

In a previous paper[1] the writers analyzed time-dependent deflections of prestressed beams tested at the University of Illinois and at the University of Florida,[2] and the observed deflections of a highway bridge under traffic in Rabat, Morocco.[3] The primary object of this discussion is to compare the deflections measured by the authors with the predictions of one of the methods described in Reference 1.

Time-dependent deformations of prestressed concrete beams are ascribed to three effects: (a) shrinkage of the concrete, defined as change in strain under no externally applied stress, (b) creep of the concrete, usually defined as change in strain (excluding shrinkage) under constant stress, and (c) relaxation of the reinforcement, defined as change in stress at constant strain. These phenomena which are not simple even in their pure defined forms, are further complicated in prestressed concrete beams. Creep occurs under varying stress and relaxation occurs under varying strain.

*Members American Concrete Institute, Research Assistant and Associate Professor, respectively, Civil Engineering Department, University of Illinois, Urbana, Ill.

Usually creep of the concrete is the most important variable affecting time-dependent deflections. Consequently, the effect of varying stress on creep must be considered. In Reference 1, it was found that the time-dependent deflection of prestressed concrete beams could be estimated with a procedure involving a combination of the rate-of-creep and superposition methods.[4,5] The basic properties of the unit creep relationship* were assumed to be independent of the variation in stress. However, the effects of the prestress, the dead load, and the applied load were computed separately using the rate-of-creep method, with time zero referring to the time of application, and then superposed at the appropriate times to obtain the net deflection.

DEFLECTION CALCULATIONS

To calculate the deflections of the beams reported by the authors, it was necessary to make assumptions about creep, shrinkage, and relaxation. The total unit creep strain for 7 years was assumed to be twice the instantaneous strain for a unit stress, the total shrinkage strain to be 0.0006 and the total relaxation loss to be 5 percent of the effective prestress. The variation of these effects with time was assumed to be described by the following expression suggested in Reference 6.

$$\varepsilon = K \ln(t + 1)$$

where ε is the fraction of the total for a given time and t represents time in days. The constant K was chosen to be 0.127 so that $\varepsilon = 1.0$ in 2600 days, the duration of the tests.

The modulus of elasticity of the concrete was assumed to be given by the following expression which is a slightly modified form of that given by Jensen:[7]

$$E_c = \frac{30{,}000}{6 + \frac{10}{f_c'}}$$

where E_c = instantaneous modulus of elasticity of concrete, kips per sq in.
f_c' = concrete strength, kips per sq in.

In the analysis of the deflections, it was assumed that the distribution of strain over the depth of the section was linear at all times and that the time-dependent variables could be considered as step functions. A numerical integration method involving the following steps was used:

1. Obtain the gross increase in creep strain by multiplying the stress in each extreme fiber at the beginning of the interval considered by the increment of unit creep strain for that interval.

2. Determine the creep strain at the level of the center of gravity of the reinforcement.

*The variation with time of creep strain for a unit stress.

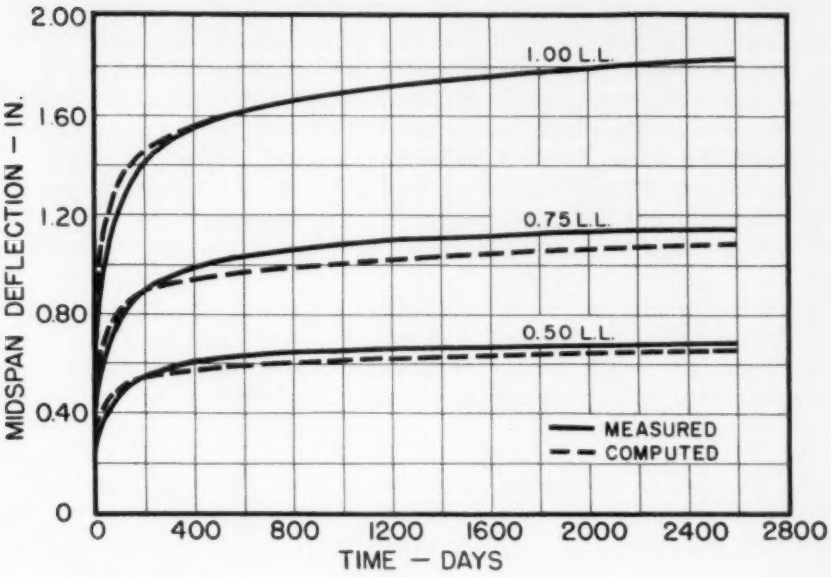

Fig. C—Comparison of measured and computed deflection curves

3. Sum the creep strain determined in Step 2 and the shrinkage strain increment for the time interval considered to obtain the total change in strain at the level of the reinforcement.

4. Determine the total loss in reinforcement stress for the interval considered by adding the loss caused by shrinkage and creep (the strain in Step 3 times the modulus of elasticity of the reinforcement) to the relaxation loss for the corresponding time interval considered.

5. Find the change in stress in the extreme fibers corresponding to the loss in steel stress. Using the short-time modulus of elasticity, E_c, of the concrete, determine the corresponding change in strain.

6. Determine the net change in creep strain in the extreme fibers by finding the algebraic difference between the gross change in strain (Step 1) and the change in strain caused by the change in stress in the reinforcement (Step 5).

7. Find the stress in the extreme fibers at the end of the time interval by finding the algebraic difference between the initial stress and the change in stress determined in Step 5.

The change in curvature during each time interval considered was calculated from the strains determined in Step 6. Since it was desired to obtain the complete deflection versus time curve, the time intervals were chosen such that the total creep was divided into 25 equal increments. The creep strains caused by the prestress, the dead load, and the applied load were considered separately. In the computations for the beam carrying the full live load, the modulus of rupture of the concrete was assumed to be one tenth of the cylinder compressive

strength. This resulted in a computed crack height of about 0.5 in. immediately on application of load and about 1 in. at the end of the test.

The computed curves are compared with those measured in Fig. C. The agreement is reasonably good throughout the duration of the test.

The method described briefly above was used to compute time-dependent deflections of the beams described in Table B. The ratios of the measured to computed deflections for the final observation time for all available tests are listed in Table C. The time-dependent properties of the materials were based on reported values, where they were available.

The fact that there is a close correlation between the computed and measured results is in itself not significant. After all, the assumptions about the relationship between total creep and instantaneous strain and the shrinkage of the concrete were made in full view of the test results. For example, if the authors' test results had indicated larger deflections, the writers would have had no qualms about assuming a higher ratio of creep to instantaneous strain. However, it is quite significant that a single set of assumptions could be used to yield good correlation between measured and computed values for all the beams tested by the authors.

It must be admitted that the method is time consuming. As a reasonable minimum, the use of this method would require four cycles of the procedure itemized previously repeated three times: once for the prestress, shrinkage, and relaxation; once for the dead load; and once for the live load. The effects of the dead and live loads had to be separated from each other since the beams were loaded 2 months after the prestress was released. In view of the errors that can be made in the estimations of the creep and shrinkage strains and the relaxation of the reinforcement, a more crude method would be more desirable if it is simpler and if it gives reasonably good results; in Prof. W. C. Huntington's words, "If we are going to be wrong, we might as well be wrong the easy way."

REDUCED MODULUS METHOD

The simplest method in considering time-dependent deflections is the "reduced modulus" method. It involves simply the reduction of the effective modulus of elasticity of the concrete by a factor, m, which is supposed to take care of the effects of creep, shrinkage, and relaxation. For uncracked sections, if the reinforcement is ignored

$$\Delta_t = m\Delta_i$$

where Δ_t = time-dependent increase in deflection
Δ_i = instantaneous deflection.

For prestressed concrete, this expression can give misleading results. Depending on the amount of prestress, the time-dependent deflection might be in the opposite sense from that of the deflection due to the applied load. This conclusion would never be drawn from the previous

TABLE B — TIME-DEPENDENT PROPERTIES OF PRESTRESSED BEAMS

Mark	Reference No.	Duration, months	Applied load	Span, ft	Beam width, in.	Beam height, in.	Depth, in.	Reinforced area, sq in.	Concrete strength, psi	Prestress, kips per sq in.
MU-1	1*	24	No	6	4	6	4.0	0.18	3760	125
MU-2	1*	24	No	6	4	6	4.0	0.18	3930	126
ML-1	1*	24	Yes	6	4	6	4.0	0.18	3800	115
ML-2	1*	24	Yes	6	4	6	4.0	0.18	3550	122
1	2†	3.5	No	25	10	12	8	1.32	6500	109
2	2†	3.5	No	25	10	12	8	1.32	6500	109
3	2†	3.1	No	25	10	12	8	1.32	6500	66
4	2†	3.1	No	25	10	12	8	1.32	6500	66
P2	3‡	21	Traffic	90.5	9.45‡	78.7	68.6	10.90	8200	140

*The prestress was released at 5 days for MU-1, MU-2, and ML-1, and 7 days for ML-2. ML-1 and ML-2 were loaded at the third points of the span a few hours after release. The nominal maximum and minimum stresses in the middle third of the span at loading were as follows: ML-1, 2000 psi compression and 152 psi tension; ML-2, 1000 psi compression and 921 psi compression. Beams were stored under constant conditions of 50 percent relative humidity and 75 F.

†Beams 1 and 2 were post-tensioned at 20 days and Beams 3 and 4 at 34 days. All beams were stored in the laboratory.

‡The beam in Reference 3 was an unsymmetrical I-section. The value given under beam width is the width of the web. The top flange was approximately 59 x 12 in. and the bottom flange approximately 22 x 8 in. All post-tensioning cables were draped in the shape of a parabola. Post-tensioning operations were begun 60 days after the beam was cast and required 7 days to complete. For the first year, deflections were measured for the beam under post-tensioning and beam dead load stresses only. The beam was then moved to the bridge site where a slab was cast on it. Deflection measurements after this time included deflection due to the weight of the slab.

TABLE C — RATIOS OF TIME-DEPENDENT DEFLECTIONS OF PRESTRESSED BEAMS

Source	Mark	Measured deflection* Δ_{meas}, in.	Computed deflection* Δ_{comp}, in.	$\frac{\Delta_{meas}}{\Delta_{comp}}$
University of Wisconsin	0.5LL	0.69	0.66	1.05
	0.75LL	1.15	1.09	1.05
	1.0LL	1.83	1.83	1.00
University of Illinois	MU-1	0.21	0.22	0.96
	MU-2	0.20	0.22	0.91
	ML-1	0.29	0.27	1.07
	ML-2	0.11	0.10	1.10
University of Florida	1	0.80	0.88	0.91
	2	0.83	0.88	0.94
	3	0.57	0.56	1.02
	4	0.62	0.56	1.10
Rabat, Morocco	P2	1.37	1.34	1.02

*Deflection measured with respect to the position of the beam before application of load for beams with applied loads and with respect to the position of the beam before prestressing for all others.

expression. To apply to prestressed concrete members it may be modified as follows:

$$\Delta_t = m(\Delta_i + \Delta_p)$$

where Δ_p = deflection caused by the prestress.

This expression may still lead to gross errors. If the beam is loaded several months after it is prestressed, it may deflect with time in the direction of the load even if the load is small. (This was the case for the beams with half live load reported by the authors. If these beams had been loaded immediately after prestressing, the time-dependent deflections might have been upward.) Furthermore, the time-dependent effects caused by the prestress depend on the level of prestress. If the prestress is high, the relative effect of a given loss in stress is not as important as when the prestress is low. Thus, to obtain reasonable trends using the reduced modulus method, it is necessary to make so many modifications that it loses its simplicity, which is its only advantage.

The method described in this discussion can be programed for a digital computer. Whenever the basic creep, shrinkage, and relaxation versus time relationships are not established under conditions similar to those for the structure in question, it would be desirable and possible to compute the deflections for the upper and lower bounds of these relationships. For preliminary design, the seriousness of time-dependent deflections can be estimated by comparing the deflection caused by the prestress with that caused by the permanent loads. If these two quantities are comparable, the time-dependent deflections with respect to the position of the beam before prestressing should not be objectionable.

The writers cannot deny the desirability of a simple one-step method for the prediction of the deflection of prestressed concrete beams. However, they feel that at this stage of development of our understanding of prestressed concrete, it would be unwise to adopt an oversimplified solution that does not make some provision for all of the known effects affecting the time-dependent deflections of prestressed concrete beams.

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By I. O. OLADAPO* and R. D. DAVIES†

The writers found this paper very interesting, as they had themselves recently tested two beams under sustained loads for about 4 months. The beams were as identical as possible, having been cast simultaneously from the same mix in a twin mold on a vibrating bed. The mix consisted of ⅜ in. gravel, sand, and rapid hardening portland cement; the water-cement ratio was 0.45, and the aggregate-cement ratio was 5.0. The beams were post-tensioned and grouted 10 days after casting. The prestressing wires were 0.2 in. in diameter, with an ultimate strength of about 235,000 psi; the initial prestress in the wires was 157,000 psi.

The beams were loaded 14 days after casting by applying dead load through a lever system. Fig. D shows the dimensions of the beams and the arrangement of load points. The curvature, $1/R$, was obtained by measuring the relative deflection of the central point over a 2 ft gage length within the region of constant bending moment in the central part of the beams. Beam A was given a moment of 2235 ft-lb (66.5 percent of its actual ultimate moment) and Beam B was loaded to 2835 ft-lb (81 percent of its actual ultimate moment). The loads were left on for 135 days.

In Fig. E, the dimensionless quantity d/R is plotted against the fourth root of time, as this straightens out the curves conveniently. It was found that 75 percent of the final deflection occurred by the 40th day in both cases. As one would expect, the rate of increase of creep was much greater for the beam with the larger moment. The final deflections were more than twice (2.5 times for Beam A, and 2.85 times for Beam B) the initial values. When the beams were unloaded after 135 days, most of the recovery was immediate, and almost all the rest occurred within the first day after unloading.

At 160 days, the beams were tested to failure in 40 min using the same load points. The cube strength of the concrete was then 7200 psi. The moment-curvature relationships are shown in Fig. F; the Curve C

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is an average for three beams similar to Beams A and B which had previously been tested at the same rate of loading. The three curves show that the moment-curvature relationships were considerably changed due to previous cracking under sustained loads. However, neither the ultimate moment of Beam A, nor that of Beam B differed by more than 3 percent from the average value for the three beams tested previously, this suggests that creep deformations do not affect ultimate moments.

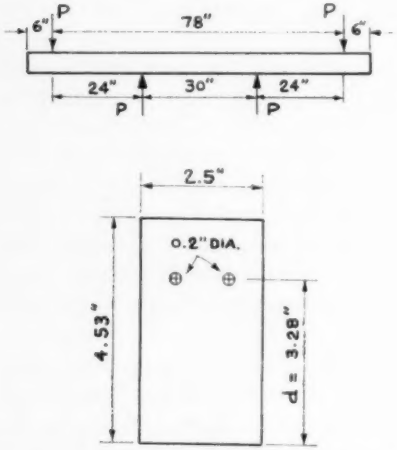

Fig. D—Beam dimensions and load arrangement

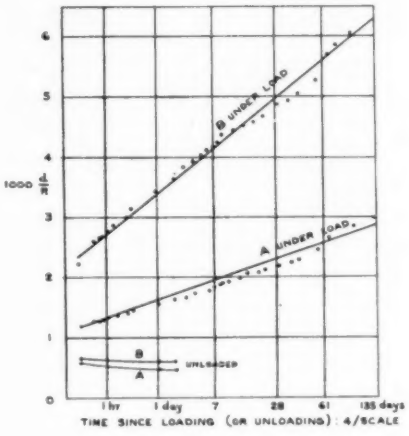

Fig. E—Variation of curvature under sustained loading

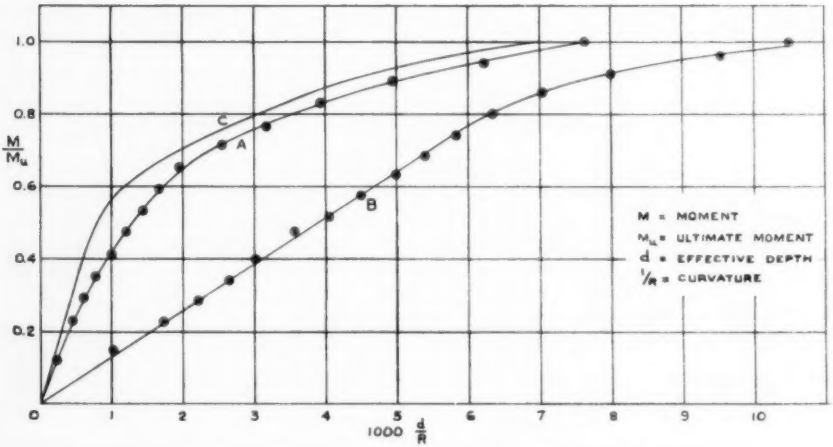

Fig. F—Moment-curvature relationship of beams tested to failure

AUTHORS' CLOSURE

The authors thank the discussers for the additional valuable information they have provided. The discussion by Messrs. Corley and Sozen on the problems of calculating time-dependent deflection of prestressed concrete beams provides an important supplement to the paper. The confirmation of the authors' results and conclusions reported in the discussions of Messrs. Branson, Davies, Oladapo, and P. W. Abeles is gratifying.

In answer to questions raised by Dr. Abeles, the beams were made in July 1952 and were given their initial 4000-lb loading at about 28 days. The total time required for the 4000 lb load test was about 15 min per beam. The beams were placed in position and subjected to the long-time loading during August at 2 months. They were kept under constant load in the basement of a large building with no airconditioning. Consequently, the relative humidities were high during each summer and low during each winter. The load was removed from the beams in November 1959 and recovery readings were taken over 64 days.

Discussion of a paper by Joseph Glucklich and Ori Ishai:

Rheological Behavior of Hardened Cement Paste under Low Stresses*

By S. CONSTANTINESCU-CĂTUNESTI, ROBERT L'HERMITE, and AUTHORS

By S. CONSTANTINESCU-CĂTUNESTI†

The authors suggest that the rheological response of cement stone to load depends on hygrometric conditions to such a degree, as to make the resulting quantitative differences seem like differences of quality. This fact is based on use of the advanced experimental apparatus which led to ingenious conclusions. The aim of the work is to demonstrate that the free viscous flow of the cement stone might be eliminated. This means that there is no free Newtonian element in the model of the material (i.e., without a Maxwell body).

Such a theory implies that the cement stone and the concrete are materials without relaxation. The presence of a Maxwell body in the Burgers model permits the equation

$$\sigma = \sigma_0 \left[\exp \left(- \frac{t}{T_{rel}} \right) \right]$$

for $t = 0$, $\sigma = \sigma_0$ and $t \to \infty, \sigma = 0$ to contain, also, the stress relaxation.

If one admits, that stress relaxation is symbolized in the model proposed by the authors (see Fig. 8) by the Element H and a multiple K, say, between H and the first K, the following relations may be written:[8]

$$\left.\begin{array}{ll} \varepsilon_{\lambda_K} = \varepsilon_{E_K} & \lambda_K \dot{\varepsilon} = \sigma_K \\ \sigma_{\lambda_K} + \sigma_{E_K} = \sigma_{E_H} = \sigma & E_K \varepsilon_{E_K} = \sigma_{E_K} \\ \varepsilon_{E_H} + \varepsilon_{E_K} = \varepsilon & E_H \varepsilon_{E_H} = \sigma_{E_H} \end{array}\right\} \quad \ldots\ldots (7)$$

After some transformation, there follows:

$$\frac{E_H \lambda_K}{E_H + E_K} \dot{\varepsilon} + \frac{E_H E_K}{E_H + E_K} = \sigma + \frac{\lambda_K}{E_H + E_K} \dot{\sigma} \quad \ldots\ldots (8)$$

$$\text{with} \quad T = \frac{\lambda_K}{E_H + E_K} \quad \text{and} \quad E = \frac{E_H E_K}{E_H + E_K}$$

$$TE_H \dot{\varepsilon} + E\varepsilon = \sigma + T\dot{\sigma} \quad \ldots\ldots (9)$$

*ACI JOURNAL, *Proceedings* V. 57, No. 8, Feb. 1961, p. 947.

†Professor of Reinforced Concrete, Institutul de Constructii, Bucharest, Rumania.

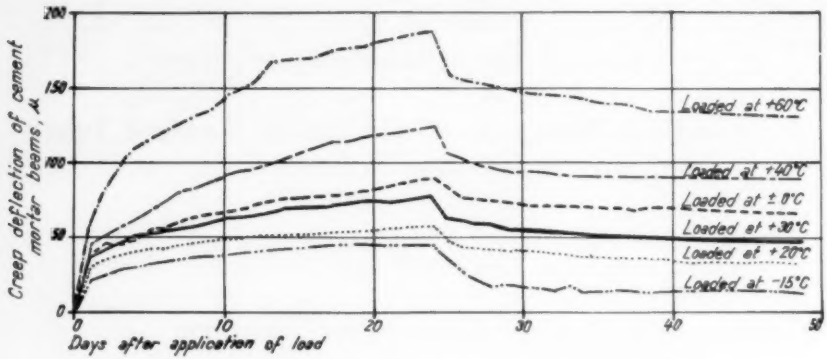

Fig. A—Creep deflection and creep recovery curves

One obtains a constant deformation $\varepsilon = \varepsilon_0$ and $\dot{\varepsilon} = 0$.

Hence,

$$E\varepsilon_0 = \sigma + T\dot{\sigma} \text{ and } \sigma = E\varepsilon_0 \left[1 - \exp\left(-\frac{t}{T}\right)\right] \quad \text{.....(10)}$$

When $t \to \infty$, Eq. (10) becomes $\sigma = E\varepsilon_0 =$ constant. This means that cement stone or concrete are building materials without stress relaxation!

On the other hand, the authors think that the question of whether there exists a free Newtonian element or not, will be settled in the future, when the nature of the secondary (irrecoverable) creep will be established. The authors seem to believe that cement stone stored and loaded only in water does not have secondary creep. However, tests performed by Hansen[9] point to the presence of secondary creep under the most severe conditions of storage in water. Indeed, in the words of Hansen, all samples used in his tests were initially cured for 6 months under water at a temperature of 35 C to avoid any further hydration during the period of sustained loading. Moreover, all samples were covered with four layers of completely impermeable plastic membrane and loaded in thermostatic baths of liquid paraffin at different temperatures, to avoid any moisture exchange with the surroundings during the tests. Hansen's tests are similar to those of the authors with the specimens being tested in flexure. The difference is that Hansen used samples of concrete. The specimens were loaded for 24 days and at unloading the creep recovery was followed for another 24 days. The deflection caused by warping was measured on companion specimens without load, and all creep curves were corrected for warping as well as for instantaneous elastic deflection. The hydration process did not continue during the 48 days of testing, even for the beams which were exposed to 60 C.

The creep deflection curves as well as the creep recovery curves are shown in Fig. A, which show an important secondary creep. The ratio of reversible to nonreversible creep is not infinite for any curve.

In water or in moist curing, the ability of cement stone to resist tensile stress is markedly different from concrete. Pickett[10] and others proved that there was a functional difference between cement stone or concrete when subjected to water storage and when subjected to dry storage conditions. This difference becomes evident under loads. It was shown[9] that the strain in the extreme tensioned fiber under dry storage is about four times as large as the strain in the extreme compressed fiber. We have calculated that the flexural deflections of the authors' beams are six times less than the deflections in the samples (with the same geometry and manufacture, but tested in dry conditions) of Reiner[2] (Fig. 11.3 and 11.4). This produces an increase in creep deflection of the beam. The flexural deformation curve of cement stone (or concrete) stored in real conditions (i.e., the conditions of design), differ qualitatively from the flexural deformation curve of cement stone (or concrete) stored in water or in a high moisture atmosphere. In a previous work, the first author pointed out that in the case of bending, slender beams fail completely on the appearance of the first crack. It was shown[9] that in the first 2 hr of loading the instantaneous deflection of dry-stored beams tripled due to the formation of new cracks (separate from those formed at the loading) but did not break. The cracks are not easily seen with the naked eye.

As far as the structural engineer is concerned dry concrete is an entirely different material than concrete stored in water.

In the last years we arrived at the conclusion that cement stone or concrete under load suffer a progressive microcracking which is the cause of the irrecoverable deformation. This obliges one to add an element to every model to represent this deformation, which we will call plastic deformation. The authors agree with such an opinion, using the K element. As a basic model the writer has proposed[11] Fig. B by maintaining the Burgers model and adding a Paez element.[12] The Paez element would symbolize the plastic behavior of cement stone and concrete, being able to represent complex situations arising from the com-

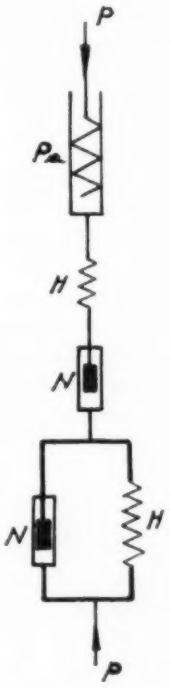

Fig B—Basic model

bination of the spring factor H and the friction on the cylinder. On removal of the load, the spring will be released and will partially restore the element. The friction constant should be time-dependent. The nonrecoverable deformation due to internal partial readjustments within the material will be represented by the same element.

It should be remembered that, with complex models, like those proposed by the authors or by Freudenthal,[4] it is difficult to maintain their generality because of the great number of constants (eight or more) which must be determined. Even in the model proposed by the discusser, there are still five constants, but they are not related to each other and hence can only be determined by laboratory measurements.

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By **ROBERT L'HERMITE***

This paper suggests a number of observations, which I present herewith.

What the authors regard as reversible creep is in fact an unloading strain, measured as such. It is the reaction of the unloaded elastic skeleton on the plastic and viscous phase that surrounds it. Experiment shows that this phenomenon occurs without variation in weight, hence without recovery of humidity. It shows that the strain has indeed the form

$$\varepsilon = \varepsilon_i \left[1 - \exp\left(\frac{-t}{T_x}\right)\right]$$

as the authors[13] indicate and that the corresponding viscosity factor increases with age and with the degree of hydration of the cement.

Although the distinction between reversible creep and irreversible creep is of interest, the writer believes there is a different approach which consists in separating three parts.

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1. A continuous and irreversible viscous part ε_v such that:

$$\frac{d\varepsilon_v}{dt} = \dot{A}(t) \quad \varepsilon_v = A(t) \quad A(t) = a\,\frac{t}{F(a,H)}$$

where $F(a, H)$ is a function that increases with the age a of the concrete and its degree of hydration H.

2. A reversible visco-elastic part ε_e such that

$$\frac{d\varepsilon_e}{dt} = \dot{B}(t)\,(\varepsilon_{me} - \varepsilon_e), \quad \varepsilon_e = \varepsilon_{me} \exp\,[-B(t)]$$

where ε_{me} is the maximum visco-elastic strain and $B\ (t)$ is a function similar to $A\,(t)$ to within one factor.

3. A feebly reversible elasto-plastic part ε_p such that

$$\frac{d\varepsilon_p}{dt} = \dot{C}(t)\,(\varepsilon_{mp} - \varepsilon_p), \quad \varepsilon_p = \varepsilon_{mp} \exp\,[-C(t)]$$

It seems that viscous creep has its point of origin in the cement phase in the course of hydration and that the viscosity is dependent on the speed of hydration. The placing in water of a test piece previously kept under load in a dry atmosphere leads to a rapid resumption of creep; the viscosity factor diminishes when hydration is resumed. A concrete cured for a long time in water before loading has only a feeble or negligible viscous creep. If H represents the volume of the hydrated phase, we may say that

$$\varepsilon = \frac{\alpha(t)}{H} + \frac{\beta(t)}{H}\,\frac{dH}{dt}$$

The first term representing the plastic strain and the second the viscous strain. This is a simple representation, more phenomenological than mathematical, which is intended only to show the meaning of the phenomena. It makes it possible to explain that creep is more quickly stabilized in a damp atmosphere than in dry air.

The difficulty of interpreting creep phenomena arises from the fact that we are confronted with a body whose structure is in continual evolution. It is because of this that shrinkage and creep have been confused, without it being clear whether these were two distinct phenomena or two aspects of a single phenomenon. In the first place, we know that there may be creep in the absence of shrinkage, in a sealed atmosphere for example, which certain authors have called basic creep.

We know that for concretes habitually tested at early age creep depands on the hygrometry of the curing medium, and that it becomes greater as the relative humidity θ diminishes. Here creep obeys a law similar to that of shrinkage. Finally, when a cylinder, that has been loaded for a long time in dry curing, is immersed in water a swelling is produced which becomes greater as the load increases, yet does not have a higher reabsorption of water measured by weight.

At first we seem to be confronted with apparently contradictory facts. But if the viscous creep is separated from the plastic creep, a corner of the veil that conceals the truth can be lifted. The viscous creep, in fact, continues during the whole period of hydration and it is possible to demonstrate[13] that this creep increases at the same time as $(1 - \theta)$. For a completely hydrated concrete, if the viscous creep disappears, the influence of the humidity θ must likewise disappear. To show this the writer took samples of concrete cylinders cured for several years in water and placed them under load with a relative humidity, $\theta = 50$ and 75 percent, and finally in water. The creep measured after a few months was then the same, no matter what the value of the humidity θ. The writer thus separated viscous creep from plastic creep. We now understand why, on immersion after unloading, the swelling becomes greater as the applied load is increased. It is because the resumption of hydration has diminished the viscosity, and the increase in swelling that we observe is the continuation and the resumption of the elastic return accompanied by a decrease in viscosity and by an increase in plasticity.

The pursuit of research on creep in concrete should, in the writer's opinion, take hydration into account. It should include the study of strain on completely hydrated concretes to separate the plastic creep of a solid in which the structure is fixed from the evolving creep which follows the strain of this structure.

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AUTHORS' CLOSURE

The authors are not certain whether the divergence of opinion with Professor L'Hermite with regard to the nature of the creep of concrete is merely due to different terminologies or due to a basic difference of opinion. When concrete is spoken of as being a "viscous" material, the authors interpret it as meaning that the material, *minus* the contained free water, flows like a viscous liquid, and if Professor L'Hermite actually holds this view then indeed there exists a basic difference of opinion, for the authors consider concrete a solid. But when he states that ". . . it is possible to demonstrate that this (viscous)* creep increases at the same time as $(1 - \theta)$." (θ being the ambient humidity), a statement clearly supporting the seepage theory and opposing such viscous theories as defined herein, the authors suspect that in essence the writer shares

*The authors' parentheses.

their view that what viscosity there is in concrete is that of the water within its pores.

The experimental evidence provided by Professor L'Hermite is enlightening indeed, but seems to be adequately explained by the authors' theory which, following some recent developments, they hope to present in a future issue of the JOURNAL. For example, when "a concrete cured for a long time in water before loading has only a feeble or negligible viscous (presumably meaning nonrecoverable) "creep" can be explained by the fact that under such conditions hydration in the capillaries is almost complete, the material is devoid of capillaries and creep is due almost entirely to flow of gel water, which is a reversible process.

Another fact stated by Professor L'Hermite, that "when a cylinder, that has been loaded for a long time in dry curing is immersed in water a swelling is produced which becomes greater as the load increases, yet does not have a higher reabsorption of water" may be explained as follows: In a dry atmosphere, capillaries but not gel pores are almost dry. When a load is applied water is expelled from the gel pores to the capillaries, and the larger the load the larger the amount of this water. On immersion in water, the gradient is reversed and water re-enters the gel from the capillaries and from the surrounding water, which causes swelling depending on the previous load. This does not necessarily imply an increase in weight in proportion to the swelling, as the re-entry of water from capillaries to gel (i.e., without absorption from outside) is sufficient to cause swelling, as was proved by the authors experimentally and will be published soon.

Finally, the apparently crushing evidence that completely hydrated specimens as opposed to early age specimens, do not show sensitivity to ambient humidity with regard to their creep, is also understandable. In early age concrete an appreciable volume (depending on the original water-cement ratio) is taken up by capillary voids. Capillary water is not firmly held within the material compared with gel water. Hence, differences in ambient humidity make a lot of difference with regard to whether the capillaries are saturated or empty, and this in turn affects the creep. In old concretes, on the other hand, (with an original water-cement ratio not more than 0.4) capillaries do not exist any more in most cases and the entire water content is gel water. Now this water is held firmly by the hygroscopic gel and variations in humidity between 50, 75, and 100 percent would make little difference with regard to flow induced by the applied load. Moreover, this fact is borne out by the phenomenon, already commented on, that such creep (i.e., in old concretes) is almost entirely reversible.

In conclusion, the authors fully agree with Professor L'Hermite that the future study of the creep of concrete should take hydration into account, but consider that perhaps the best way to separate the creep of the final structure from that due to evolving hydration would be to re-

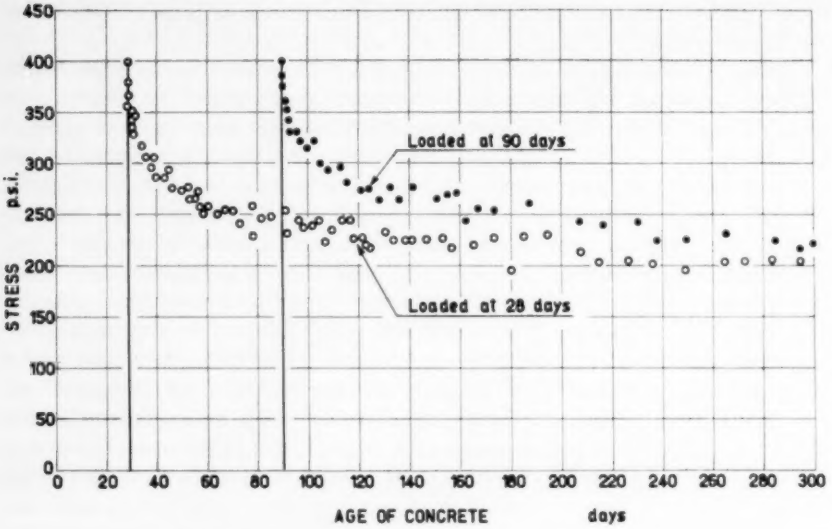

Fig. C—Stress relaxation versus age of concrete

move, at a certain stage of hydration, all the evaporable water existing at that stage, thereby arresting all further hydration.

The authors are grateful to Professor Constantinescu-Cătunesti for his comments, particularly for giving them the opportunity to add relevant information collected since the discussed paper appeared in print.

With regard to his comment concerning stress relaxation, however, the authors must disagree completely. A material comprising a Hookean element and a Kelvin body in series (a so-called "standard solid") will most certainly exhibit stress relaxation. It is true that such relaxation, as distinct from that of a Maxwell body, will not be complete, but has anybody ever claimed that in concrete stresses relax completely? Using the notation employed by Professor Constantinescu-Cătunesti, it can be easily shown that at $t = 0$ the stress is

$$\sigma_0 = \varepsilon E_H \qquad (11)$$

while at $t = \infty$

$$\sigma_\infty = \varepsilon \frac{E_H E_K}{E_H + E_K} \qquad (12)$$

which clearly shows a decrease in stress with time. The complete function of stress relaxation under constant strain is

$$\sigma = e^{-t/T} \varepsilon E_H + \varepsilon \frac{E_H E_K}{E_H + E_K} (1 - e^{-t/T}) \qquad (13)$$

where $T = \lambda/(E_H + E_K)$ is the relaxation time.

The authors have not carried out relaxation experiments, but Hansen[9] presents such results, reproduced here in Fig. C. These curves show that stress relaxation in concrete is *not* complete. If concrete was a Burgers body as suggested by Professors Constantinescu-Cătunesti and Hansen, then obviously relaxation would be complete. Indeed, if we assume equal moduli for the Hookean and the Kelvin elements (i.e., $E_H = E_K$), Eq. (12) will become

$$\sigma_\infty = \varepsilon \frac{E_H}{2} = \frac{\sigma_0}{2} \quad \text{.....(14)}$$

which is nearly what we see in Hansen's experimental results.

SECONDARY CREEP

With regard to the second point raised by the discusser, i.e., the nature of the secondary (nonrecoverable) creep, the authors have meanwhile conducted creep-recovery tests under water and, in agreement with Hansen,[9] found that nonrecoverable creep does indeed take place under water. However, the authors wish to point out a few facts with regard to Hansen's results, since they were brought up by the discusser in Fig. A:

1. Hansen did not conduct his creep and creep-recovery tests under water. His specimens were cured under water, but prior to loading they were taken out and, quoting Hansen's description, "covered with four layers of a completely impermeable plastic membrane. . . .". Now, if the authors have understood this statement correctly, the specimens were not coated with a solution of a plastic material which, on drying would form a skin adhering to the specimen (as done by the first author in 1957)[1] but wrapped in a sheet of a plastic membrane. The authors have already shown in the paper under discussion that while the former method is almost equivalent to full saturation (providing the specimens were saturated prior to coating), the latter method permits escape of water to the outside surface of the specimen, the inside of the membrane and the intervening air, thus favoring higher water loss and consequently higher nonrecoverable creep. In underwater testing, creep recovery is much higher, as unbroken continuity is maintained between the water in the specimen and that in the bath.

2. Hansen had his specimens creep for 24 days and recover for 24 days. From the authors' experience, recovery, to be practically complete, should last at least three times as long as creep.

3. Hansen assumes that after 6 months of underwater curing, no further hydration should occur during the period under load, and therefore what nonrecoverable creep there might be cannot be attributed to hydration. This assumption is probably justified for mechanically-undisturbed specimens. As soon, however, as a load is applied to a speci-

men that has apparently (as judged from strength determinations) reached old age, rejuvenation takes place due to renewed movement of ion-bearing liquids. This movement disturbs the chemical balances established previously and results in renewed hydration oriented in accordance with the applied load. In all probability, such renewed hydration will not be reflected in a strength increase (strength being a poor indicator of *oriented* hydration) and in any case will certainly not be detected by strength determinations of control specimens not subjected to load. The authors believe that renewed hydration cannot be ruled out as a cause of nonrecoverable creep, even in quite old concretes.

Had Hansen conducted his experiments under water instead of wrapping his specimens in plastic, and had he waited longer for recovery to be completed, he might have observed much smaller nonrecoverable creep, but nevertheless he would have observed such creep. For, as mentioned before, the authors have also observed such creep when testing submerged specimens and waiting for recovery to end. However, they do not interpret this result as Hansen and Professor Constantinsecu-Cătunesti do, namely that the nonrecoverable creep is a viscous flow, for they now strongly believe that hydration is the cause of this creep (and to a smaller degree also nonreversible cracking). In the paper under discussion the authors say that the existence or nonexistence of nonreversible creep in underwater testing will decide the issue. They now agree that such an experiment would not be conclusive, and indeed the issue was decided, in their opinion, by another experiment which they hope to describe in detail in a future issue of the JOURNAL. Briefly, this experiment consisted in the removal of various proportions of the evaporable water (W_e) from the material and measuring the corresponding rates of creep. They were able to show that the rate of creep was a decreasing function of the proportion of W_e left in the material, attaining linearity at low values of W_e and that for $W_e = 0$ the material did not creep at all. They regard this as conclusive evidence that the nonrecoverable creep is not a viscous flow.

The authors are also in agreement with Professor Constantinescu-Cătunesti's remarks concerning the effect of the moisture conditions on cracking deformation. Preliminary experiments with a view to clarifying the mechanism of this deformation seem to indicate a far-reaching influence of wetness of the outside surface of the specimen on its strength and deformation. It is presumed that this influence is manifested in affecting the surface tension of the material. There are indications, however, that it is not the state of humidity in the specimen which induces the cracking deformation, but rather the change in this state, resulting in either inflow or outflow of water through the outside surface. Sealed specimens, when suddenly exposed to the atmosphere (whether of higher or lower relative humidity than their own) will thus undergo excessive deformation. In concrete in most cases, such deforma-

tion will soon be stabilized and resume its pre-exposure rate; in hardened cement paste, on the other hand, it often leads to fracture. The authors hope to be able to say more about this point in the near future.

Professor Constantinescu-Cătunesti proposes to use Paez'[12] element for representing the cracking deformation. The authors find this unsuitable for the following reasons: (a) Results reported by the first author[1] and others show that the time-independent deformation in hardened cement paste is perfectly linear. Paez' element, on the other hand, involves a nonlinear instantaneous component. (b) Paez' element would have to be installed in parallel with a dashpot to allow for the fact that cracking progresses with time when the specimen is under sustained load. In such a case, however, it will not account for the fact that for short periods under load the entire deformation (including that due to cracking) is recoverable.

Discussion of a Report by ACI Committee 662:

Formwork for Concrete*

By MERLE E. KERSTEN† and CHARLES F. PECK, JR.‡

The writers have studied the report with a great deal of interest and in most instances heartily concur with the committee's findings. There are, however, a few points which we think need either correction or clarification as to intent.

Part 1—Safety Precautions

"The 4 x 4-in. shores should not be used as reshores; 6 x 6 in. is the minimum recommended size." The writers believe that this is an unwarranted restriction which will unnecessarily add to the cost of reinforced concrete construction. Metal and 4 x 4 in. wood shores have been used to reshore millions of square feet of concrete joist and solid slab floor construction. When the shores are properly used there have been no reshore failures. It would be much more practical to emphasize the need for thorough analysis of the shoring and reshoring and to allow the contractor to use the size and type of reshoring best adapted to his operation, so long as he does it safely and considers all the necessary factors mentioned in the committee's report.

Part 2—Engineer-architect specification

The writers would emphasize the obligation of the engineer-architect to provide more complete camber specifications. With longer spans becoming more popular both the short-time elastic and long-time plastic deflections become greater and must be properly compensated for in the formwork. Instances have arisen when an effort was made to hold the contractor responsible for an uneven floor when in reality the amount of camber specified was in error.

The writers also believe that the engineer-architect should indicate the design loads for each part of the structure. This is required in Section 102 of the ACI Building Code (ACI 318-56) but does not seem to be common practice. When the contractor knows the design loads, he can readily determine the construction loads which he can safely impose on the completed portion of the structure.

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*ACI JOURNAL, *Proceedings* V. 57, No. 9, Mar. 1961, p. 993.

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Part 3—General requirements for concrete formwork

In Section 103(b) there are three points that need clarification. The writers are of the opinion that the manner of exclusion of "design analysis" for some structures in Paragraph 1 could be misinterpreted and should be clarified. Apparently it was the intent of the committee that a "rule of thumb" design would suffice for some of the smaller structures. However, just as much damage can occur to a structure and the risk to men working on the deck is just as great whether the placing operation is taking place 15 ft above the ground or 15 ft above a cured floor 20 stories up.

The second point is Paragraph 2 in which the writers think that proved patented splicing devices, such as the Ellis clamp, should definitely be permitted when used with proper care. The limitation on the number of spliced shores might be misinterpreted, unnecessarily prohibiting the use of workable devices.

The third point is the skepticism placed on horizontal joints in Paragraph 5. The writers see no reason why this type of joint should be excluded if conservative design stresses are used. Although they agree with the committee that they should be held to a minimum, they are concerned that in some instances economical and sound formwork might be excluded through a misunderstanding of the specification. Therefore, the writers suggest that the second two sentences of the paragraph be omitted.

The writers appreciate the effort which the committee has put into this report and hope that it will serve as a positive step toward better formwork for concrete construction. They believe, however, that the need for good rational analysis of formwork is great and should be emphasized in the code. Arbitrary regulations which unduly penalize the careful contractor should not be included.

Discussion of a paper by Franco Levi:

Work of the European Concrete Committee*

By P. COHEN; LUIS P. SÁENZ, IGNACIO MARTÍN, and RAFAEL TAMARGO; JOSEPH TAUB and A. M. NEVILLE; and AUTHOR

By P. COHEN†

In the resolution on cracking the value f_s obviously must have some lower limit, below which the published crack relations do not apply. The writer wonders whether Dr. Levi could throw some light on this point. It is often important to know what crack widths can be expected for values of f_s as low as 850 kg per sq cm. The writer is aware that for steel stresses of this magnitude, cracking is of a far less predictable nature than for the higher stresses usually considered in crack-width investigations, but wishes to draw attention to the fact that water-retaining structures such as reservoirs and tanks are often designed with low working stresses in the steel. The British Code[1] for these structures limits steel stresses to 12,000 psi (850 kg per sq cm), in the expectation that crack widths should they appear, will be sufficiently small so as not to endanger the safety of the structure or to cause significant leakage.

The writer has proposed[2] that a more economical method, based on limiting crack width should be tried in the design of water-retaining structures, making use of the ability of concrete to heal itself under suitable conditions. It would be interesting to know whether CEB has considered the possibility of using crack-width design methods for the design of water-retaining structures.

The writer wishes to inquire of Dr. Levi if CEB examined the methods of crack-width design in use in the Soviet Union as given in their reinforced concrete Code.[3] These methods have been in general design use since 1955, and by now there should be a great deal of data from which it should be possible to draw conclusions on the reliability of the design methods. The Soviet code gives the following relation for members in direct tension (transforming the relation into the form given in the CEB resolution):

$$0.25 \frac{D}{P_1} f_s \leq \alpha$$

*ACI JOURNAL, *Proceedings* V. 57, No. 9, Mar. 1961, p. 1041.

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For "structures calling for the highest durability requirements, subjected to repeated dynamic loads, or exposed to outside atmospheric action, or in conditions of high air humidity (relative humidity higher than 60 percent), as well as in silos and chimneys," α is given as 4200 for smooth bars, and 8400 for deformed bars.

The author is to be commended for the concise and lucid manner in which he has presented this valuable and interesting material.

REFERENCES

1. "Design and Construction of Reinforced and Prestressed Concrete Structures for the Storage of Water and Other Aqueous Liquids," British Standard Code of Practice, CP 2007, 1960.

2. Cohen, P., "Crack-Width Design of Axially-Tensioned Water-Retaining Reinforced Concrete Structures," *Transactions*, South African Institution of Civil Engineers, Oct. 1961.

3. "Instructions for the Design of Sections of Reinforced Concrete Structural Elements," Ministry of Industrial Construction, USSR.

By LUIS P. SÁENZ, IGNACIO MARTÍN, and RAFAEL TAMARGO*

The procedure adopted by the European Concrete Committee for evaluating the loss of strength in a column due to its slenderness is acceptable. Nevertheless, we deem it convenient to comment on some of the simplifications in the methods of calculations used, to obtain expressions which are not strictly in accordance with reality.

In the first place, to determine the additional moment which is superimposed in the column due to its slenderness use is made of the Madrid parabola:

$$\frac{\sigma}{\sigma_o} = \frac{\varepsilon}{\varepsilon_o}\left(2 - \frac{\varepsilon}{\varepsilon_o}\right)$$

This is an expression based on the nonlinear behavior of concrete represented by the ascending branch of the parabola considered. If the behavior of the concrete is represented by a curve of the type shown in Fig. A, its equation would be defined by the group of minimum conditions which it would be required to satisfy. These conditions are:

1. Considering only the action of the external loading, zero load will produce zero deformation, in other words $\sigma = 0$, $\varepsilon = 0$ corresponds to a point on the curve.

2. The tangent to the curve at the origin (0,0) shall have a slope whose value is equal to the longitudinal initial elastic modulus

$$\frac{d\sigma}{d\varepsilon} = E_o \quad \text{when} \quad \varepsilon = 0$$

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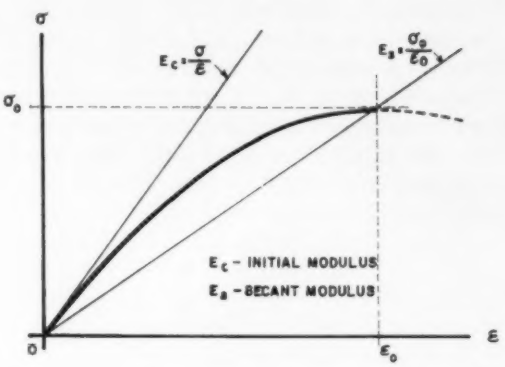
Fig. A — Ultimate strength behavior of concrete based on Madrid equation



3. The vertex of the curve will pass through the point of maximum stress which defines the quality and the strength of the concrete for short-time loading. The vertex coordinates are $(\varepsilon_o, \sigma_o)$.

4. Since the point with the above coordinates is a maximum of the curve it follows that

$$\frac{d\sigma}{d\varepsilon} = 0 \quad \text{when} \quad \varepsilon = \varepsilon_o$$

The equation with the least number of terms that may satisfy these conditions will be a third degree parabola having the following general expression

$$\sigma = A + B\varepsilon + C\varepsilon^2 + D\varepsilon^3 \qquad (20)$$

Solving the system for the conditions previously stated, furnishes the following values for the coefficients of the general equation [Eq. (21)].

$$A = 0 \qquad (21)$$

$$B = E_o \qquad (22)$$

$$C = \frac{6\sigma_o - 4E_o\varepsilon_o}{2\varepsilon_o^2} \qquad (23)$$

$$D = \frac{E_o\varepsilon_o - 2\sigma_o}{\varepsilon_o^3} \qquad (24)$$

Substituting Eq. (22), (23), (24), and (25), in Eq. (20) we have

$$\sigma = E_o\varepsilon + \frac{6\sigma - 4E_o\varepsilon_o}{2\varepsilon_o^2}\varepsilon^2 + \frac{E_o\varepsilon_o - 2\sigma_o}{\varepsilon_o^3}\varepsilon^3 \qquad (25)$$

Multiplying and dividing the second member of the equation by σ_o, and since $\sigma_o/\varepsilon_o = E_s$ is the secant elastic modulus (see Fig. A), we have

$$\sigma = \sigma_o\left[\frac{E_o}{E_s}\frac{\varepsilon}{\varepsilon_o} + \left(3 - 2\frac{E_o}{E_s}\right)\left(\frac{\varepsilon}{\varepsilon_o}\right)^2 + \left(\frac{E_o}{E_s} - 2\right)\left(\frac{\varepsilon}{\varepsilon_o}\right)^3\right] \qquad (26)$$

Although it does not come within the scope of this discussion, an analysis of the values which correspond to the modular ratio E_c/E_s was made by checking the behavior curves of numerous tests with concrete of different strengths, during the development of a general theory for reinforced concrete in flexure. That analysis will be submitted in due time for consideration by ACI. The analysis shows that for concrete strengths from 1500 to 3000 psi, $E_c/E_s \approx 2$. Substituting this value in Eq. (26) gives us

$$\sigma = \sigma_o \left[2 \frac{\varepsilon}{\varepsilon_o} - \left(\frac{\varepsilon}{\varepsilon_o} \right)^2 \right]$$

which is the same as Eq. (5), p. 1055.

$$\frac{\sigma}{\sigma_o} = \frac{\varepsilon}{\varepsilon_o} \left(2 - \frac{\varepsilon}{\varepsilon_o} \right) \qquad (5)$$

This was the equation adopted in Madrid.

From the foregoing it is evident that the Madrid equation is applicable without appreciable error solely for concrete strengths of 1500 to 3000 psi, and that it deviates from reality as the strength of the concrete increases. This is an important point, because the demand of high strength concrete for compression members is increasing.

The second comment refers to the fact that the European Concrete Committee using Eq. (5), obtains the deflection at the center of the span from the following equation (p. 1056):

$$\delta = \frac{t}{2\sqrt{3}} \frac{\sigma_o}{\sigma_E} \sqrt{1 - \frac{\sigma_o}{\sigma_o}}$$

Here a new simplification is introduced by assuming the value of the square root as unity for slender columns, which requires that $\sigma_c/\sigma_o = 0$.

In first place it could be convenient to define what is understood by slender columns. Two of the writers of this discussion, Sáenz and Martín, have made an investigation of tests performed with columns with flat ends and different ratios up to a geometrical slenderness ratio of 43, (150 mechanical slenderness ratio). The tests and conclusions are included in a paper presented to the ACI entitled "Tests of Reinforced Concrete Columns with High Slenderness Ratios," and pending publication.

This investigation reveals that even for geometrical slenderness ratios as high as 43, the value of the ratio σ_c/σ_o is greater than 0.60, which will produce an approximate value of 0.63 for the square root which is supposed to be unity. This introduces an error of about 37 percent in the value of the superimposed moment M_c for the conditions stated previously.

Another debatable point, in the determination of the moment due to the effect of buckling, is the value assigned to E_c (p. 1058). There it

is suggested that the value of E_c may be taken as approximately equal to $1000\, f_c'$.

The value indicated is, with an acceptable margin of error, only applicable to concretes with a quality of 2000 to 4000 psi, but this would give a large error in the case of the high quality concretes which are being used with increasing frequency. It might be convenient to select an expression that would give a more reliable value of E_c, which without complicating the method adopted by the Committee, would, nevertheless, offer greater precision regardless of the quality of the concrete employed.

An additional comment is offered on the precision of the criteria adopted by the European Committee for buckling in establishing a comparison between the values calculated with the formulas indicated in the paper, and the results obtained from tests performed with columns having geometrical slenderness ratios as high as 43, which were investigated by Sáenz and Martín as previously mentioned.

To obtain the ultimate load of the columns (P_u) the following formula was used:

$$P_u = \frac{M_u}{e + \dfrac{M_c}{P_u}} = \frac{0.375\, bd^2 f_c'}{e + 0.3\, t \dfrac{\sigma_c}{\sigma_E}}$$

From the characteristics of the column section tested, the amount and position of their reinforcement and end conditions, Euler's characteristic stress $\sigma_E = K \pi^2 E_c/\lambda^2$ becomes

$$\sigma_E = 1.75 \frac{\pi^2 E_c}{\left(3.44 \dfrac{l}{t}\right)^2}$$

The results are given in Table A for comparison.

From Table A, it is seen that the method recommended by the European Concrete Committee gives values sufficiently close to the experimental results. The analytical and experimental values coincide fairly well, in spite of the fact that the simplifications used do not represent reality in some cases. This can be attributed to the following:

1. The qualities of concrete used in the test were mostly 2500 to 5000 psi. Within this range $E_c = 1000\, f_c'$ and $E_c/E_s = 2$ do not have a considerable error, as has been pointed out previously when referring to the simplifications.

2. The error in the value of the additional moment due to buckling, M_c, as mentioned before, introduces a minor error in the determination of P_u, because its terms appear as an additive term in the denominator of that equation.

3. Finally, since the terms used in the equation that determine P_u, though based on a criterion and values that are not exact, but which produce errors that compensate each other, the final results are acceptable.

To sum up, the writers maintain the same criterion which was expressed by the Austrian delegation, who favored a simpler method of calculation. Due to this criterion, the authors of the investigation of the

tests with slender columns already described in detail, have found that it is possible to predict the critical strength of columns with flat ends and axial loads using a Rankine type general expression

$$P_u = \frac{P_o}{1 + C\left(\frac{l}{t} - m\right)^n}$$

where $P_o = 0.85\, f_c' bt + f_y\,(A_s + A_s')$.

The expression derived from the experiments is

$$P_u = \frac{P_o}{1 + 0.00017\left(\frac{l}{t} - 6\right)^{2.3}}$$

which for practical use is expressed as

$$P_u = \frac{P_o}{1 + 0.001\left(\frac{l}{t} - 6\right)^2}$$

When the expression derived from the experiments is used to make a comparison with the results of the tests, it furnishes values of P_u (calc)/P_u (test) with a minimum of 0.96 and a maximum of 1.04.

TABLE A—COMPARISON OF P_U (TEST) WITH P_U (CALC) FOR REINFORCED CONCRETE COLUMNS

Column group	f_c' psi	l/t	P_u (test), kips	P_u (calc), kips	$\frac{P_u \text{ (calc)}}{P_u \text{ (test)}}$
26D	3350	25	54.5	51.0	0.94
23D	2840	25	57.0	45.0	0.79
3E	4560	25	63.0	70.0	1.11
31D	5050	25	83.5	80.5	0.97
					Avg 0.95
27D	3560	30	46.4	48.0	1.04
24D	3020	30	47.0	42.0	0.90
1E	4930	30	72.0	66.3	0.92
10E	4630	30	82.5	64.5	0.78
					Avg 0.91
29D	3190	35	40.0	37.9	0.95
30D	3050	35	42.0	37.4	0.89
2E	5360	35	56.0	63.6	1.14
20D	4160	35	53.0	51.0	0.96
					Avg 0.985
14E	4250	40	52.0	46.0	0.89
6E	2780	40	33.0	29.2	0.89
15E	2910	40	39.0	31.4	0.80
5E	5590	40	54.0	58.5	1.08
					Avg 0.915
21F	2640	43	34.4	25.5	0.74
28F	4380	43	48.7	43.5	0.89
					Avg 0.815

By J. TAUB* and A. M. NEVILLE†

Dr. Levi gives in this paper a summary of the conclusions which have so far been approved by CEB. The writers are especially interested in resolutions concerning the shearing stress of reinforced concrete members.

For sections submitted to high shear and low bending moment it is assumed by CEB that the ultimate behavior of a beam is represented with good approximation by the classical truss analogy. Many tests performed on reinforced concrete beams seem, however, not to confirm this assumption.

In Fig. B is represented a part of a beam without web reinforcement located to the left of a diagonal tension crack as a free body. It is known that once the crack reaches the level of the tension steel in such a beam the distribution of stresses in the tension steel and in the concrete along the beam does not follow the distribution of external moments. This is called a redistribution of internal stresses.

In Fig. C (left) is represented a beam the web reinforcement of which consists of vertical stirrups. Under some load a diagonal tension crack forms in the beam as shown in this figure. Considering again the part of the beam located to the left of this crack as a free body it is seen that the internal forces along Line *abc* act here in the same way as in a beam without web reinforcement. Therefore, a redistribution of internal stresses takes place in this beam in the same way as in beams without web reinforcement. The total shear acting on the free body is taken by the concrete at the upper end of the diagonal tension crack and by the tension steel at its lower end.

An increase in the applied load causes the diagonal tension crack to extend further as shown in Fig. C (right). The total shear is now taken by (a) the concrete at the upper end of the diagonal tension crack, (b) the stirrups, and (c) the tension steel at its lower end.

With further increase of the load on the beam the stress in the stirrups increases until it reaches the yield point. This is the beginning of the plastic range. The force taken by the stirrups now remains constant and the further increase in shear is taken by the concrete

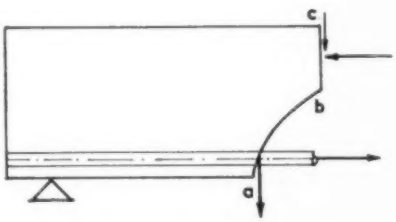


Fig. B—Redistribution of internal stresses in beams without web reinforcement

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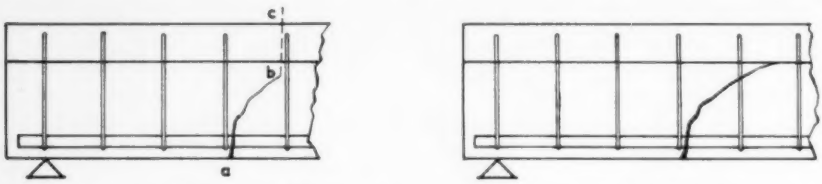

Fig. C — Redistribution of internal stresses in beams with web reinforcement

at the upper end of the diagonal tension crack and by the tension steel. In this way a second redistribution of internal stresses now takes place.

In Fig. C it has been assumed that the diagonal tension crack formed as a development of a flexural crack. Let us now consider the case where the crack opens first in the middepth of the beam. In this case the diagonal tension that acted before along this crack is now taken by the concrete at both ends of the crack and by the stirrups, if any, that are crossed by the crack. As the tensile strength of the concrete is low the crack rapidly extends towards the compression zone and the tension reinforcement, and a redistribution of internal stresses takes place as described before.

After the redistribution of internal stresses the shear strength of a reinforced concrete beam depends on the resistance of the compression zone above the diagonal tension crack and the tension reinforcement at the lower end of the crack, respectively.

Two types of shear failure are described in the conclusions of CEB: (1) destruction by compression of the inclined diagonals of concrete in the web, and (2) rupture of the stirrups.

No cases of rupture of the stirrups are known to the writers. The yield of the stirrups leads to the redistribution of internal stresses as described before. If the load on the beam is increased beyond the yield point of the stirrups the shear failure may occur in one of three ways, viz.,

(a) By crushing of concrete at the top of the diagonal tension crack, i.e., shear compression failure

(b) By the destruction of the tension zone between the lower end of the diagonal tension crack and the beam support, i.e., shear tension failure

(c) By the opening of a flat-slope crack up to the top surface of the beam, i.e., diagonal tension failure

The writers presented some time ago an extensive study on the resistance to shear of reinforced concrete beams.*

*Taub, J., and Neville, A. M., "Resistance to Shear of Reinforced Concrete Beams", ACI JOURNAL, *Proceedings* V. 57: "Part 1—Beams without Web Reinforcement," No. 2, Aug. 1960, p. 193; "Part 2—Beams with Vertical Stirrups," Sept. 1960, p. 315; "Part 3—Beams with Bent-Up Bars," Oct. 1960, p. 433; "Part 4—Behavior of Beams with Different Types of Web Reinforcement," Nov. 1960, p. 517; "Part 5—Anchorage and Bond," Dec. 1960, p. 715.

AUTHOR'S CLOSURE

Because of the interest of the contributions to the discussion which was sent to me after publication of the article in which I briefly summarized the results obtained by the European Committee in the session held in Vienna in April, 1959, it would have been preferable if my responses could have been written with the collaboration of the reporters of the commissions on buckling, cracking, and shear. Lack of time and the necessity of maintaining my remarks in narrow limits obliged me to respond directly. Nevertheless, I will bring the observations of Messrs. Sáenz, Martín, and Tamargo; Taub and Neville; and Cohen to the attention of the experts of CEB for deeper examination.

The remarks of Messrs. Sáenz, Martín, and Tamargo on the formula for calculating buckling failure adopted by the CEB testify to a thorough knowledge of the question. In particular the observations on the character of the expressions adopted by CEB to represent the law of deformation of the concrete, for the evaluation of the modulus of elasticity, and for the calculation of the deflection of the end-loaded beam are certainly valuable. The approximations have been discussed on another occasion within the European Committee and have been accepted (with the object of simplifying the solution), under the reservation of experimental verification of the results of the method of calculation. The results of this verification, totaling 143 tests, were published in CEB *Bulletin* No. 17; it has resulted in a mean value of the ratio of experimental failure load to theoretical failure load equal to 1.16, which is the same order of magnitude found by Mr. Sáenz and his collaborators. The CEB estimates that, considering the characteristic dispersion of the results of buckling tests, the experimental verification can be considered quite satisfactory and that it justifies the approximations adopted. Conforming to the general orientation of the work of the Committee, it has also been admitted that the formula of Aas-Jakobsen, which is based on a rational interpretation of the phenomena, was preferable to purely empirical expressions where the possibilities of extension are always necessarily uncertain. It is worthwhile to remark that the CEB method is applicable not only to columns, but equally to prismatic beams of all types and to two-dimensional elements.

We add that, in the CEB session held at Monaco in January, 1961, the formula of Aas-Jakobsen was slightly modified in adopting a rectangular stress block. The new formula for the superimposed moment is

$$M_s = \frac{P(t + e)E_s}{3300\,\sigma_s}$$

where e indicates the initial eccentricity measured with respect to the center of gravity of the section. This value was compared with a large number of experimental results in CEB *Bulletin* No. 31. The results

obtained confirm that the CEB method furnishes an approximation sufficing for practical needs.

In conclusion, I wish to remark that Mr. Sáenz and his collaborators have incorrectly interpreted the position of the Austrian delegation with respect to the CEB. In effect the Austrian delegation did not propose the development of "a simpler method of calculation;" in reality Jäger and his colleagues objected that the formulas of Aas-Jakobsen reverted the problem of buckling to a problem of composite bending and wished to replace them by tables or charts established on the basis of a theory of instability taking into account the numerous parameters affecting the phenomena.

As to the remark of Mr. Cohen, it is correct that the formulas concerning cracking adopted at Vienna did not consider the case of lightly reinforced members for which the force transmitted to the concrete by the reinforcement remains less than the tensile strength of the surrounding concrete. Under such conditions, the crack is propagated by accidental phenomena, such as shrinkage, noncontinuous concreting, or variations of temperature. This is notably the case of reservoirs as mentioned by Mr. Cohen.

The problem was, however, considered in the recent session at Monaco, in the course of which the Vienna resolutions were submitted to revision. The "cracking" commission of the CEB is considering assuming the reinforcement in lightly reinforced members to act like bars fixed between two blocks of concrete, separated by the eventual crack. Conforming to this point of view, the commission has proposed that the crack opening should be considered a variant proportional to the square of the stress in the steel, to the diameter of the bars and inversely proportional to the bond coefficient, but independent of the percentage reinforcement. Consequently, for a given quality of bond and tolerances of crack openings, the maximum permissible diameter of bars (in the case of low percentages) would vary proportionally with the tensile strength of the concrete, and inversely with the square of the stress in the steel at the crack.

As to the comparison of the CEB propositions with the formulas of the current Soviet code, it is interesting to note that, at the time of the recent plenary session in Monaco, the commission considered presenting the cracking conditions earlier adopted, in the simplified form.

$$\frac{D}{p_i} f_s \leq a_o$$

This simplified expression, valid in the case of normal percentages corresponding to a systematic and complete development of the crack pattern, is essentially in a form analogous to the Soviet formula. With regard to the practical values of a_o to be introduced in the practical

CEB recommendations, they are now being derived from a study of experimental results.

The author wishes to thank Messrs. Taub and Neville for their interesting contribution. It is certain that the distinction between sections subjected to a predominant shearing force and sections subjected principally to bending has an arbitrary character and that it does not completely agree with a scientific analysis of the phenomena. It is no less certain that the classical truss analogy does not represent exactly the real redistribution of internal stresses during diagonal cracking. From this point of view it is interesting to note, that in the course of our most recent work sessions, several CEB delegates called attention to the internally statically indeterminate character of the shear problem.

Statical indeterminacy implies the necessity of taking into account conditions of compatibility of deformations, and not solely the equilibrium conditions of the forces. In effect, if one neglects the conditions of compatibility, an indeterminate number of solutions will be found mathematically. The extension of the classical truss theory (initially proposed in the case of predominant shearing forces) and the new theory of Paez (proposed in the case of predominant bending moments) thus constitute two particular solutions among the infinite number possible. This proves that there remains a fundamental indetermination and that the statically indeterminate system, which a diagonally cracked beam actually is, can be correctly solved only by considering the conditions of compatibility of the deformations.

Because of this complexity of the problem, CEB has held, while awaiting indispensable scientific studies, to the statistical interpretation of existing tests, following the proposition of Torroja. This conclusion was adopted during the recent plenary session in Monaco, in January 1961.

This practical formula which conforms to the analysis of the tests assumes that the resistance to shear is obtained by adding the contribution of the transverse reinforcement, calculated by the truss analogy, to the contribution of the concrete. This latter force takes into account the resistance of the compression zone of concrete under shearing stress, and also the influence of the "excess" longitudinal tensile reinforcement above that required to resist the actual bending moment. In effect, this excess longitudinal reinforcement contributes strongly to reducing the opening and the development of cracks, which augments the importance of the uncracked zone of the beam and, at the same time, the participation of this zone in resisting shear. The conclusions reached at Monaco may be found in CEB *Information Bulletin* No. 32. It is interesting to

note that the provisional formulas adopted by the CEB give results near to those which are being studied by ACI.

On p. 1068, Professor Inge Lyse's name was inadvertently omitted from the list of members of Commission I — Tentative Recommendations and Commission V — Shear.

Discussion of a paper by Robert G. Mathey and David Watstein:

Investigation of Bond in Beam and Pull-Out Specimens with High-Yield-Strength Deformed Bars*

By RAYMUNDO RIVERA V. and AUTHORS

By RAYMUNDO RIVERA V.†

With reference to the authors' article, I wish to inform our readers that in the Strength of Materials Laboratory of the College of Civil Engineering of the University of Nueve Leon, tests were made to determine the relative bond strength of high-yield-strength deformed bars for reinforcing concrete.

The procedure followed was identical to that recommended by ACI Committee 208, Bond Stress.[3]

The tests were made using a ⅞ in. diameter high-yield-strength steel bar with a proportional limit of 4180 kg per sq cm (59,500 psi), a maximum tensile strength of 7910 kg per sq cm (113,000 psi), and an elongation of 15.5 percent in 8 in.

A hydraulic testing machine of 60,000 kg capacity was used. It was necessary to adapt the machine to a series of attachments to meet the requirements established by ACI Committee 208.

The velocity of the applied load during the tests was 2000 lb per min and readings were taken at intervals of 500 kg without interrupting the velocity of application of the load. Careful records were made of the slips and deformations as recommended by Committee 208, and also of the location of each crack and the corresponding load.

The writer found a divergence with the results described in the last paragraph of p. 1083, deduced from Table 5.

Fig. A was made using the data from Table 5 and four other graphs for each length of embedment, viz., 7, 10.5, 14, and 17 in., all converted into the metric system. From Fig. A it was deduced that the loaded-end slip is independent of the length of embedment as asserted by the authors. However, our tests indicate that there is a marked effect when the lengths of embedment are changed for the same size bar.

*ACI JOURNAL, *Proceedings* V. 57, No. 9, Mar. 1961, p. 1071.
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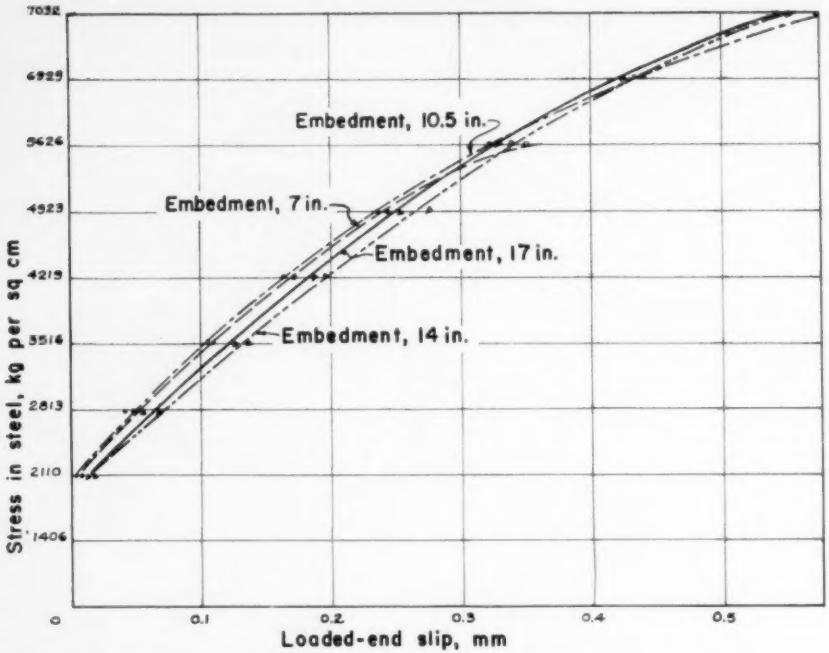

Fig. A—Stress-slip relationship for beams with varying embedments

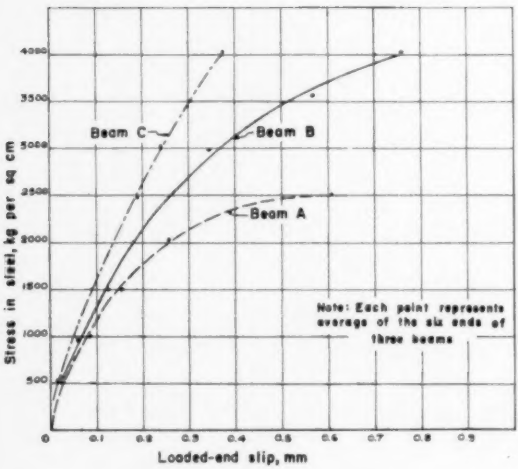
Beam C
Beam B
Beam A
Note: Each point represents average of the six ends of three beams
Stress in steel, kg per sq cm
Loaded-end slip, mm

Fig. B—Stress-slip relationship for cast beams in straight position

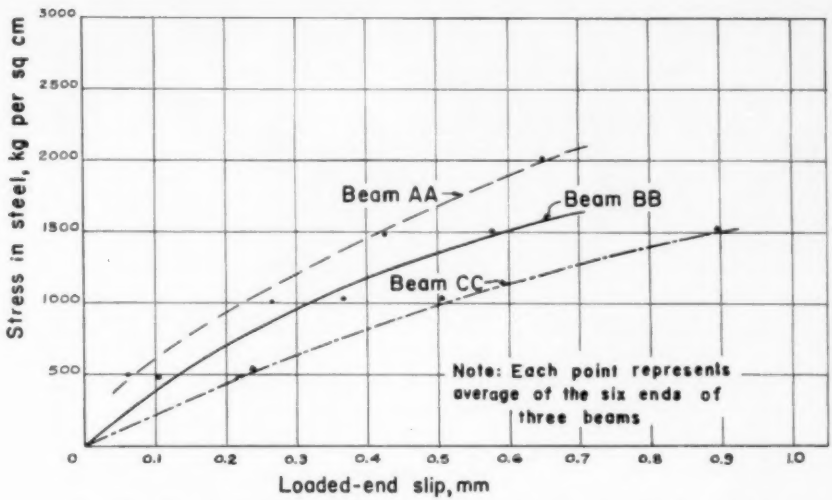

Fig. C—Stress-slip relationship for cast beams in inverted position

Fig. B shows the results obtained with nine cast beams in a straight position. The three beams (A) have a length of embedment of 8 in., the three beams (B) have a 12-in. embedment, and the three beams (C) have a 16-in. embedment. Each point on the graph represents the average of observations made on the six ends of the three beams.

Fig. C shows the results obtained with nine cast beams in the inverse position with the same lengths of 8, 12, and 16 in. Here again the effect of changing the length of embedment for the same size bar may be noted.

The writer found also that the curves representing the relationships between the computed bond stresses and the corresponding computed and observed steel stresses approach each other as the load increases as asserted by the authors.

Because of financial restrictions it has not been possible to obtain more conclusions from the results of the tests on the 18 beams, but we will notify our readers of any forthcoming deductions.

AUTHOR'S CLOSURE

The apparent discrepancy between the stress-loaded-end slip relationships as determined by the two investigations for different lengths of embedment of a given size bar may possibly be attributed to the differences in the extent of transverse cracking in the shear spans of the beam specimens. It must be pointed out that in the absence of transverse cracking the stress-loaded-end slip relationship in a beam specimen approaches that in a pull-out specimen. Mains[5] demonstrated that

beams and pull-out specimens yield comparable results when the length of a pull-out is comparable to the distance between the free end of the bar in a beam and the nearest crack within the shear span.

With the foregoing in mind, it is of interest to note that the stress-loaded-end slip relationship for Nuevo Leon beams containing #7 bottom bars embedded 12 in. agreed remarkably well with the NBS pull-out specimens containing #8 bars. From Nuevo Leon's Fig. C it can be seen that the stress-loaded-end slip relationship for beams having 8- and 12-in. embedments agree fairly well with each other up to a steel stress of 30,000 psi; the point on the curve corresponding to a steel stress of 35,000 psi for the beams having an 8-in. embedment probably represents a large free-end slip. It was also noted that there was no appreciable difference in the stress-loaded-end slip relationship for the range of steel stresses common to the NBS beams containing #8 bars embedded 7 and 14 in. and the Nuevo Leon beams containing #7 bars embedded 14 in. From these observations it might be presumed that the Nuevo Leon beams having 8- and 12-in. embedments did not develop transverse cracks in the shear span whereas the beams having 14-in. embedments did develop transverse cracks. It will be recalled that the authors reported that all beams with #8 bars developed cracks within the shear span.

Discussion of a paper by Lewis H. Tuthill, Robert F. Adams, Shelly N. Bailey and Ronald W. Smith:

A Case of Abnormally Slow Hardening Concrete for Tunnel Lining*

By ROBERT A. KINZIE, JR., L. LIBERTHSON, K. E. PALMER, BAILEY TREMPER, and AUTHORS

By ROBERT A. KINZIE, JR.†

Cement has been an inexpensive but most useful product. Its great utility has in large part been due to wide general acceptance of and compliance with ASTM standards. Compliance with the standards has given the consumer the certainty of success.

During the last few years many proprietary compounds have appeared on the market. They are sold to accelerate or delay setting time, increase plasticity, increase strength, decrease required mixing water, etc. Many of them will do what is claimed but the side effects are not always known or if known, have not always been desirable.

There have been cases where a cement to be used successfully with an admixture must have an SO_3 content too high to meet the requirements of other large users of cement.

These conflicting desires of large consumers and the increasing sale of admixtures without the safeguard of proper standards presents a real problem. At this time it seems as though the cement companies can either make a standard cement with the burden on those who wish to make or use admixtures to be sure they are compatible with all standard cements or the cement companies can go into the drugstore business making a cement for each consumer and admixture. Either course is possible but the latter is too costly to be acceptable without considerable discussion.

It does put the burden on the cement companies who are financially responsible to make a product which can be satisfactory with any compound the user may care to use.

By L. LIBERTHSON‡

The authors' paper is a significant contribution to the technology of concrete admixtures if only because it focuses attention on a subject which many engineers would not hesitate to state has more to it than meets the eye.

*ACI JOURNAL, *Proceedings* V. 57, No. 9, Mar. 1961, p. 1091.
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It has long been recognized that the safe use of lignin sulfonic admixture in concrete requires the establishment of rigid adherence to safe dosages and great care in design of the concrete to make certain that the admixture is compatible with the cement used. During the years prior to this recognition the records of at least one manufacturer of such admixtures were marked with embarrassing frequency, with instances in which concrete failed to develop one or another of its normal hydraulic properties because of the presence of lignin sulfonic bodies.

Whatever the true explanation for the abnormally slow hardening described by the authors, it seems to this writer that if the reported deviations in SO_3 content of portland cement are indeed critical, they would tend at least to discourage, if not altogether proscribe, the use of lignin sulfonic type admixtures. Certainly the observations of the authors' constitute ample basis for a rigorous re-evaluation of the role of set retarding admixtures in general, and of the lignin sulfonic type in particular. The entire subject indeed seems to be one in which the more we know, the more there is still to learn.

Among the questions suggested, to perhaps more than one reader, are the following:

1. Would the same deviation in SO_3 content of portland cement exert a similar, perhaps equally drastic effect on the hydration of the cement if an admixture of the carboxylic acid type were used?

2. If the latter type functions satisfactorily and independently of the SO_3 factor, would not a specification writer or an engineer naturally regard the lignin sulfonic type as risky, and prefer the carboxylic type as safe?

3. Is it not possible that the SO_3 content of the admixture itself is at least as critical as the SO_3 content of the portland cement?

4. The admixtures in question are dispersed in the form of a relatively dilute solution. The period of storage is usually indeterminate. However, they are known to function as nutrient media for many varieties of bacterial and fermentative micro-organisms, among which sulfur metabolizing bacillae may predominate; it would seem legitimate to inquire into the extent to which such a factor might contribute to the failure of a concrete to develop normal hydraulic properties.

Undoubtedly other questions could be, and I hope have been formulated.

By K. E. PALMER*

The authors of this paper should be commended for their thorough effort in studying job conditions and materials at the time a concreting difficulty was discovered. In most other instances in which field troubles

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are encountered, samples of the concrete components actually in use are not obtained so that a laboratory study can be made with them.

Our laboratory tests have confirmed the general conclusion of this paper, namely, that some cements which set and develop strength normally, will be severely retarded when used with certain admixtures. In most cases, but not in all, we find that the severe retardation is removed by an increase in sulfate content. Control of this factor is complicated by the fact that different brands of admixtures or even different shipments of the same brand can produce varying degrees of retardation, ranging from a few hours to several days. Our investigations of cement-admixture-sulfate combinations have produced several other conclusions concerning the chemical reactions involved, as well as the practical considerations for field concreting.

An interesting and important property is shown by the curves of Fig. 8, 9, and 10. It will be noted that for cement-admixture combinations which produced the extremely slow hardening, a stiffening occurred at 2 to 4 hr after mixing which was not present in the mortars which hardened normally. This early stiffening and subsequent slow hardening are also shown in Fig. 4 of a report by Tuthill and Cordon,[2] for an increased quantity of admixture. This stiffening at 2 to 4 hr is typical for combinations showing slow hardening and evidently is caused by a change in the early hydration products from those normally produced by the cement.

Our laboratory tests have indicated that different quantities of various admixtures are required to give a significant reduction in water requirement for a cement and that different quantities of the same admixture are required with a change in cement source. In many cases a lesser quantity of admixture than that recommended by the manufacturer will provide substantially the same amount of water reduction with less retardation of strength development.

It appears that the "¼ QA" in Fig. 8 may represent an overdosage for this particular cement-admixture combination, since the cement without admixture, as well as the cement with another admixture, hardened normally. It would be important to know whether a lesser quantity of the QA product would have proved satisfactory.

The laboratory data given in this report are based on tests performed at 70 to 75 F. The tunnel concreting reported was performed during January and February and probably involved temperatures much lower than that used in the laboratory work. This may have intensified the retardation. Tuthill and Cordon,[2] show that the rate of hardening is delayed by a factor 2, for a temperature reduction from 75 to 50 F. In our laboratory tests this temperature reduction has given a much greater factor of delay for some cement-admixture combinations.

It appears that many factors influence the interaction between portland cement and water-reducing admixtures. These reactions are not

completely understood, and it is impossible to predict the behavior of all cements with all available admixtures. This variability is reported in the Tuthill and Cordon paper as well as in many of the papers included in ASTM *Special Technical Publication* No. 266.[4]

Our research investigations have indicated that admixtures react chemically with the alumina-containing phases of the cement and also that rate of silicate hydration is significantly reduced for several days, even in some instances in which severe retardation does not occur. It also appears that the admixture-aluminate reaction consumes sulfate, and for this reason larger quantities of total sulfate are necessary for optimum properties of the cement with respect to strength development and minimum drying shrinkage. The quantity of additional sulfate required for each cement-admixture reaction cannot be precisely evaluated with the present state of knowledge of the physico-chemical reactions which occur.

In the manufacture of portland cement the addition of gypsum is governed by a minimum requirement for proper retardation and by a maximum specification limit. This maximum is now set at 2.5 percent SO_3 for Type II cements. This paper reports test work at 2.4 percent SO_3 to secure a satisfactory hardening rate with one cement-admixture combination. Our laboratory tests have indicated that many cements require still higher SO_3 contents for optimum results if they are used with the manufacturer's recommended quantity of some admixtures. To minimize the possibility of field difficulties in projects which involve the use of various admixtures, it would be necessary either to delete or increase the SO_3 specification limit for the cement.

Another physical phenomenon which involves the interaction of each particular admixture-cement combination is the development of a rapid stiffening which sometimes occurs within a few minutes after the concrete has been mixed. This stiffening, sometimes referred to as rapid slump loss, is mentioned on p. 54 of the Wallace and Ore paper included in Reference 4 and is described on p. 111 in the Tuthill, Adams, and Hemme paper in the same publication.

We have found that the intensity of this stiffening varies with the quantity and brand of admixture used. Also, in some instances an increase in sulfate content has produced this stiffening in a cement-admixture combination in which it does not occur at the lower sulfate level. In these cases cited, the cements had no false setting properties at either sulfate level.

In summary, the minimum sulfate requirement for prevention of excessive retardation by an admixture varies for each combination, and an excess of sulfate over this requirement can produce undesirable slump loss as well as increased drying shrinkage. To secure the best possible concrete when it is desired to use admixtures, a laboratory investiga-

tion should be made with the specific concrete components under conditions designed to simulate the field concreting conditions. This would determine the optimum quantity of admixture to secure the desired change in water reduction, acceleration or retardation, consistent with all the other requirements for satisfactory placement, curing, and ultimate use of the concrete.

By BAILEY TREMPER*

The authors are to be commended for reporting the difficulties experienced with slow hardening of concrete containing a normal dosage of lignin-based retarder. It is fortunate indeed that personal injury or loss of life was not involved. It is not difficult to visualize a serious accident if falsework were removed from a structure or multistory building before the concrete had hardened.

The authors allude to other instances of abnormally slow hardening when a retarding admixture had been used. Polivka and Klein[6] have warned:

> With some retarding admixtures in combination with a low-alkali, low C_3A cement, the usual dosage may result in undesirable high retardation.

Tremper[7] showed, using drying shrinkage tests, that a lignin retarder produced entirely different results with variations in SO_3 content in the cement.

The writer has subsequently found a delay in hardening up to 3 days when the authors' retarder QA at ¼ lb of solids per sack of cement was used with a cement from the same mill as the authors' AC unless the SO_3 was at least equal to its optimum value. It was also found that the admixture increased the percentage of SO_3 required for greatest strength of the cement.

The optimum SO_3 content for minimum drying shrinkage (which is essentially the same value required for highest strength) has been found to vary among Type II, low-alkali cements produced in California from about 1.25 to 2.50 percent.

The mere use of a specification requiring a minimum numerical value of SO_3 content that is sufficiently high to assure normal hardening with all cements to be used with a lignin retarder could result in a severe overdose of gypsum in those cements of low optimum. An overdose of 1 percent in SO_3 usually results in detrimental expansions in the presence of water.[8] The autoclave expansion test of itself does not afford a sufficient safeguard against the effect of excess gypsum.

If the use of retarder is specified for the work, it appears to be essential to incorporate some type of specification control for the cement to be used. That the needed type of control for the cement is not simple should be evident from the above discussion.

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If the use of a retarder is not required, but is permitted, such permission should be given only with a definite understanding that the contractor shall assume full responsibility for the results obtained.

The authors conclude the paper with the statement

> There is no question but that the Type II cement in this case must have an SO_3 content well over 2 percent to be sure of avoiding greviously slow setting when lignin-base admixtures are used.

To the writer, this solution is not entirely adequate even for cement from the one mill to which it is intended to apply. It does not provide any safeguard against a decision by the manufacturer to so alter the composition or particle size distribution of the cement, within specification limits, that the optimum SO_3 content is reduced to the neighborhood of 1.25 percent. In such an event, the use of an SO_3 content well above 2 percent could result in detrimental long-time expansion of the concrete. Such a manufacturing change is not entirely imaginary because, for economic reasons, cement manufacturers often install new kilns and grinding equipment or change the source of supply of raw materials.

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6. Polivka, M., and Klein, A., "Effect of Water-Reducing Admixtures and Set-Retarding Admixtures as Influenced by Portland Cement Composition," *Special Technical Publication* No. 266, ASTM, 1959, pp. 124-139.

7. Tremper, B., discussion of "Structural and Lean Mass Concrete as Affected by Water-Reducing, Set-Retarding Agents," *Special Technical Publication* No. 266, ASTM, 1959, pp. 94-96.

8. ASTM Committee C-1 on Cement, "Short-Time Tests of Mortars for Controlling SO_3 in Portland Cement at Optimum Value," *Proceedings*, ASTM, V. 59, 1959, pp. 349-368.

AUTHORS' CLOSURE

The authors appreciate the four discussions which their paper has received and recognize the individual concerns which prompted them. Mr. Kinzie and Mr. Palmer are concerned with a situation that reflects on the uniformity of cement production and that requires closer control on the addition and perhaps character and quality of gypsum. Mr. Tremper is concerned with effects of possible shifts from the optimum amount of SO_3 and Mr. Liberthson apparently wishes to cast doubt on the inexpensive lignin base admixtures in favor of admixtures of another type.

The reader is reminded that in these cases of abnormally slow hardening, variation in the admixture could not have been a factor since it was dispensed from job supplies of several thousand gallons of a factory solution of the material, which had thorough mixing in shipping and

transferring, and by test at various ages, has never shown variation outside normal variations in testing. This clearly places the cause in a temporary variation in the cement. As Mr. Palmer points out, when variation results in too high an amount of SO_3, a reverse effect may occur making it possible to "walk on a 6 in. slump concrete in 10 minutes," as this was described by field engineers. Mr. Tremper reminds us that slow hardening can happen with more than the one brand of cement and this has been noted in field experience. But since these variations do occur in cements, and there is no change in the admixture, the imperative need for assured uniformity of cement is evident. The need for better uniformity of some cements from the standpoint of strength has been mentioned by others.

With this necessity for limiting extremes in the amount of SO_3, we feel that the optimum amount of SO_3 from the standpoint of strength and shrinkage will more often than not, and more often than otherwise, be supplied. With increasing recognition of the benefits and economy of using these admixtures, customers are likely to urge and we expect that cement manufacturers will take the necessary care in controlling the addition of gypsum, and that such improved precision will in most cases also result in a content of SO_3 that is close to optimum when the admixture is used. Some tests with the admixture indicate a different optimum value than without it and that there would be few cases when it would be outside the limits required for normal hardening.

Since the SO_3 content of a lignin admixture in normal dosage could increase the SO_3 content of the cement only a small amount, probably not more than 0.04, it is difficult to agree with Mr. Liberthson that it could be critical. Also the influence of micro-organisms is not a factor in the admixture used because it is treated with an inhibiting factor. Material so treated has been stored several years and has shown no evidence of supporting such growth.

To us it seems this experience and these helpful comments on its causes re-emphasize the value of establishing the performance characteristics of the combination of materials to be used in job concrete. It is recognized this often cannot be done, but for this reason it would seem worthwhile for cement producers to establish for their own protection, the safe limits of SO_3 in cement from their raw materials, with the commonly available lignin-base admixtures.

It is our hope that publication of this experience will cause no one to forego the benefits of these admixtures. We have continued to use them since that time with satisfaction and expect to continue to do so. Occasional abnormalities may occur but we have confidence that if they do, causes can be quickly corrected since something is known about them; there is little danger of serious job damage or harm to anyone

since hardening is now usually verified before shores are removed or forms are stripped; and such concrete will harden to better strength than normal.

Discussion of a paper by Jerome M. Raphael:

Structural Models Evaluate Behavior of Concrete Dams*

By JAN A. VELTROP and AUTHOR

By JAN A. VELTROP†

It is very encouraging to find published in the United States such an excellent paper on an important contribution to the testing of models for an arch dam type structure. This paper clearly demonstrates the advances made by the author in a field which was all but conceded to European experts. The two major contributions of the author are the development of an integration method for dead load stress measurements and adoption of the load cell method for water load measurements under increasing intensity of loading. The author is to be congratulated on this fine accomplishment which is all the more significant as it was accomplished in a relatively short time.

The writer wishes to make a few comments and ask some questions. Using the order in which those subjects appear in the paper, these comments are as follows.

Presumably the initial shape of central arch and buttresses were given to the author. It would be of interest to discuss modifications of shape (if any) suggested by the test results. A cursory examination of water load stresses indicates much lower stress values in the upper part of the central arch than in the lower sections. This would suggest savings in concrete by decreasing the thickness of the upper arches as seen in the "downstream face" (see Fig. 14).‡

Relatively low values of compressive stresses along the "bulging" downstream face (see also Fig. 14, "intrados") indicate that this face could be "straightened" by cutting off excess concrete. This contention is further supported by the rather significant change in principal stress direction near the downstream face as compared to those further upstream. This is apparent both at intrados and extrados shown in Fig. 14. The rather low values obtained for dead load stresses in the same general area, as shown in Fig. 16, point towards the same improvement in

*ACI JOURNAL, *Proceedings* V. 57, No. 9, Mar. 1961, p. 1111.
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‡Note: Captions to Fig. 13 and 14 were interchanged in the paper—EDITOR

shape. Incidentally, has the author superimposed dead load and water load stresses? If so, could he publish the results?

On what basis did the author select the ratio 1:2:4 for the moduli of elasticity for gravity plug, mass concrete structural elements, and foundation? Is the difference of gravity and structural concrete based on cylinder tests using different mixes? Is the estimate of foundation modulus based on laboratory tests on core samples and was any reduction made for estimating in situ values for an effective foundation modulus?

The question of the importance of Poisson's ratio has been discussed several times in the technical literature on structural model testing. Knowing the relationships of similitude based on theory of elasticity, one must agree with the author that model and prototype should have the same Poisson's ratio. Opinions in the literature differ, however, on the importance of the numerical value for Poisson's ratio for practical applications. Has the author any particular study in mind which shows the effect of various values for Poisson's ratio on model studies of three dimensional statically undetermined structures?

On p. 1114 the author cites desired dimensional accuracy as the reason for his selection of the model scale 1:200. As the entire method of building the prototype and the braced fiberglass mold appears rather novel, the writer wonders whether the author's statement is based on small experimental castings. Apparently it was found that size of the model, loading mechanism, and model material would meet the requirement of sufficient response in the strain gages to enable accurate measurements. Presumably a smaller model, and therefore a larger scale, could have been used to satisfy the latter requirement.

It seems that both the criteria for dimensional accuracy and for adequate strain gage response lead to a certain physical size of the model, independently of the actual prototype dimensions. The resulting model scale is merely a derived ratio rather than one that is selected a priori. Often engineers seem to think that model scale in itself is significant.

The foundation of the model was built up from prefabricated and dried plaster slabs. How did the author establish whether this method resulted in a homogeneous foundation with constant modulus of elasticity in all points and in all directions?

The author's comments on the advantage of live load testing with gradually increasing loads are well taken. A check on the linearity of behavior of the model with intensity of loading is important. It would be of interest to know whether the author encountered any effects and corresponding difficulties with creep of the model material.

In discussing the maximum tensile stress of 165 psi due to live load the author comments that this stress would probably not occur in the prototype because of the proposed mastic joint. Does the author

consider such a tensile stress too large in this area of the dam? Perhaps the author considered that the measured stress is not the maximum, as it was measured well below the crown of the arch, as shown in Fig. 14. A larger tensile stress would be expected at the crown. On the other hand, a stress concentration may have contributed to this high tensile stress value near the re-entrant corner; a reduction in the prototype due to plastic flow and creep might be expected in that case.

The author's method of dead load testing is of twofold importance. First, in comparison with other methods a much greater accuracy can be obtained because of the larger response in the strain gages. Of even greater significance is the second point, that is the possibility to reproduce actual construction sequences and thereby test critical periods during construction. In case of a dam structure without contraction joints, which will be grouted later, this is relatively simple. If, however, the opening of contraction joints is not known or cannot be accurately predicted or when the size of openings vary with elevation due to different cooling times, the model test will have the same limitations as calculations would have.

It appears to the writer that a contradiction is implied between the author's statements concerning the desirability of a plug on p. 1112 and the need for developing strength through the crown on p. 1128. Supposedly in the latter case strength cannot be developed until concreting is above the plug. This may well lead to the conclusion that the size of the plug should be reduced (even though that may not fit requirements for its use as a cofferdam). A smaller plug just filling the deepest portion of the river would probably be enough to remove the highly stressed region at the vertex of the conical elements of the central arch.

AUTHORS' CLOSURE

In his discussion, Mr. Veltrop has posed a number of questions regarding improvement of the shape of the model arch and buttress dam described in the paper. At the outset, it should be emphasized that the shape tested was delineatd by engineers of the California Department of Water Resources as one of several proposed shapes to be evaluated for possible use at Oroville Dam. At the conclusion of the model tests, the test data along with additional field data on geology as well as cost estimates on the several designs were considered together, with the result that all proposed concrete designs were set aside, and final decision was made to construct an earth and graded gravel dam at the Oroville Dam site. Hence, although a number of improvements were suggested in modifying the shape of a second proposed model, further tests of this model were unnecessary, and all subsequent engineering studies have been directed toward the embankment structure.

Although the emphasis of the paper was on methods by which stresses in concrete dams can be determined experimentally, it is interesting to speculate along with Mr. Veltrop on possible means by which the design can be improved. The author agrees with the writer that much concrete could be saved by removing the bulge from the downstream face of the main arch, possibly giving that face the conventional dog-leg shape employed in many buttress dams. Low compressive stresses generally prevalent in this region indicate that the concrete is not working efficiently. Conversely, generally higher stresses near the re-entrant angle between the riding buttress and the extrados of the arch indicate some stress concentration due to the re-entrant angle. Some improvement could be expected by fairing the two surfaces together and eliminating the abrupt angular change. The dead load and water load stresses were superimposed to find the maximum stresses in the model, and have been published in the final report given in Reference 2.

The ratio 1:2:4 for the moduli of elasticity for gravity plug, mass concrete structural elements, and foundation was selected after evaluating the probable effective elastic modulus of concrete and rock in situ at the various locations. The gravity plug is in general a low stress element, and was to be constructed of concrete having in general half the strength and elastic modulus of the concrete in the structural elements. Evaluation of the in situ elastic modulus of the foundation was made by considering the results of a number of seismic tests performed at the site. It is considered that determining the elastic modulus from core samples is misleading in that these samples do not reflect the discontinuities generally found in natural foundations. In this respect, it might be interesting to point out that one of the major recommendations of the Seventh Congress on Large Dams at Rome in 1961 was to find improved methods for the evaluation of the mechanical properties of rock foundations in situ.

The author agrees with Mr. Veltrop that a smaller model at a larger scale could have been used to satisfy the requirements of accuracy in the strain gage measurements. However, it was considered that the special conditions of this model study, all of which was performed with student help, made it necessary to set the model scale and the method of manufacture of the model according to the limitations of the labor available. With a professional staff of competent artisans, such as are easily available in many European laboratories, the author would have been content to use a much smaller scale.

The homogeneity of the foundation built from prefabricated and dried plaster slabs was established by a series of tests. The modulus of elasticity of each individual slab was determined by a sonic test. The elastic modulus of the epoxy resin used in the bonded joints was determined to be of the same order of magnitude as that of the plaster in the found-

ation. The behavior of a number of cylinders sawed apart at various angles and rejoined with the epoxy resin was compared with that of solid plaster cylinders and the behavior was considered close enough alike so that this method could be relied on to give homogeneity in the model foundation. Finally, to the author's best knowledge, no directional properties have been attributed to plaster-celite mixtures.

Some creep was found in the model material. Since the rate of creep decreases rapidly, a time schedule was adopted for all readings so that readings taken after the creep had stabilized were found to be quite reproducible.

Under combined stress, the maximum tensile stress of 165 psi due to live load near the joint between the arch and the plug disappeared. However, tensile stresses of even greater magnitude were found at the extrados under combined live and dead loads. It is considered that these could have been made to disappear by reshaping the arch near the foundation. This would have been the next step if the model studies had been continued.

If the plug were maintained in the dimensions used in the model, strength through the crown of the arches could have been devleoped earlier by using inclined lifts rather than the horizontal lifts assumed for construction. However, the test results generally pointed out that the size of the plug might well be reduced, thus affecting even greater economy in a second model.

In conclusion, the paper showed a number of new methods that can be used to predict stresses and behavior of concrete dams. The writer's comments are valued in showing how improvements could have been made in the design tested.

Discussion of a paper by Richard W. Furlong:

Ultimate Strength of Square Columns under Biaxially Eccentric Loads*

By F. N. PANNELL and AUTHOR

By F. N. PANNELL†

It is unfortunate that Professor Furlong has confined his calculations to square sections and to multibar steel patterns, and also that he has chosen actual stress values and dimensions instead of a general non-dimensional case, as this severely restricts the applicability of the results.

The plots of deviation paths for columns with steel only in opposing faces confirm what the writer has taken to be intuitively obvious, namely that these sections are inefficient in biaxial bending except when the angle θ between the actual and major axis planes of loading is small. Where there is significant moment about the two major axes, such sections may be written off as a practical design form on three counts: they are inefficient; an accurate assessment of their failure capacity is extremely difficult; a design method is available‡ for an economic column, having unequal steel in adjacent faces.

The most efficient steel pattern is that of four bars, one at each corner, and this should be used for biaxially loaded columns wherever practicable. Such columns also have a much larger deviation ratio than multibar columns, and the author's results will not be valid for them, nor for eight-bar columns. Both these sections are particular cases of rectangular sections with equal steel in all four faces, which is a valuable family of sections for which the theoretical failure load in biaxial bending is accurately predictable.[7]

The following comments relate to this family of sections. The deviation paths which the author has calculated for such (square) columns appear to be in close agreement with the equation

$$\Delta = \Delta_p \ (\sin 2\theta)^2 \qquad (1)$$

*ACI JOURNAL, *Proceedings* V. 57, No. 9, Mar. 1961, p. 1129.

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‡Pannell, F. N., "Failure Surfaces for Members in Compression and Biaxial Bending," submitted to ACI JOURNAL and pending publication.

in which the notation of the author's Fig. 5 has been used, and Δ_p is the deviation of $\theta = 45$ deg for a square section. It would be of interest to know whether the calculated deviations did in fact vary significantly from those predicted by this equation.

Since two depth of cover ratios d_2/d have been taken, the author may also be able to give confirmation to the equation

$$M_1 = M_2 \frac{M_{b1}}{M_{b2}} \quad \text{(2)}$$

relating the moments between two columns identical in loading pattern and in form except for the d_2/d ratio. M_b denotes the balanced failure moment and M the actual failure moment.

These two equations, together with the basic interaction curves for bending about a major axis and about a diagonal for a nondimensional square section, are all that is required to develop the full interaction surface for a square biaxially loaded column. If the author had chosen a general rectangular section, he would have been able to investigate a third property of the failure surface, namely that if the quantity termed e_x' in Fig. 5 is multiplied by $\phi = M_{by}/M_{bx}$ then the resulting failure surface is transformed to that of an equivalent square section. The one set of curves is thereby made applicable to the general case. Three such sets cover all cases of the steel patterns considered.

The author's conclusions may merit reconsideration. Conclusions 1 and 3 are not universally valid. For high values of P/P_o, and for low values where the reinforcement quotient is low, columns may be somewhat more efficient when bending on a diagonal. This will occur when the deviation factor N is negative. A more valuable remark would be that for symmetrical columns bending about a diagonal (or a transformed diagonal) the section with equal steel in all faces is more efficient than any other.

Conclusion 4 is not the best that could be made in the light of available evidence. If these recommendations are applied to four-bar columns it may result in an overestimate of failure load of up to 25 percent. The approach put forward by the writer, or the variation suggested by Bresler[8] with some amendments[9] would appear to be sounder than that now advocated by Professor Furlong.

The meaning of Conclusion 5 is obscure. The author has considered square columns with steel in two opposing faces, and apparently wishes to make one of two deductions: (a) That a rectangular column with steel in opposing faces only will have a higher deviation ratio than that for an equivalent square column, or (b) that any rectangular column will have a higher deviation ratio than that of an equivalent square column.

It is not clear that his evidence permits either of these extrapolations.

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8. Bresler, Boris, "Design Criteria for Reinforced Columns under Axial Load and Biaxial Bending," ACI JOURNAL, *Proceedings* V. 57, No. 5, Nov. 1960, pp. 481-490.

9. Pannell, F. N., Discussion of Reference 8, ACI JOURNAL, *Proceedings* V. 57, No. 12, June 1960, pp. 1632-1635.

AUTHORS' CLOSURE

The author wishes to express appreciation to Mr. Pannell not only for his comments regarding some general aspects of skew bending behavior, but also for his cordial and cooperative attitude during the past 2 years.

Although the presentation of a set of interaction surfaces applicable to a wide range of specific, practical cases was not the purpose nor intention of the paper, Mr. Pannell's remarks regarding the limited scope of the data presented are appropriate. The stated purpose of the paper "to describe the ultimate theoretical capacity of square, tied columns to carry biaxially eccentric loads" was accomplished. Sufficient data were presented to display characteristics common to square columns, excluding the four-bar column. That which is intuitively obvious was demonstrated in some cases, but nonetheless was judged worthy of inclusion for nonspecialists on the subject.

The cross-hatched portion of Fig. A shows the observed variation of the ratio Δ/Δ_p for various angles θ as derived from Fig. 6 through 11. Eq. (1) of Mr. Pannell's discussion does fall quite well within the boundaries of the cross-hatched region. However, linear interpolation of a family of interaction curves, such as those of Fig. 6 through 11 would be almost as accurate. The author is able to give confirmation to Eq. (2), For concrete cover ratios of 0.05 and 0.125, variations as high as 10 per-

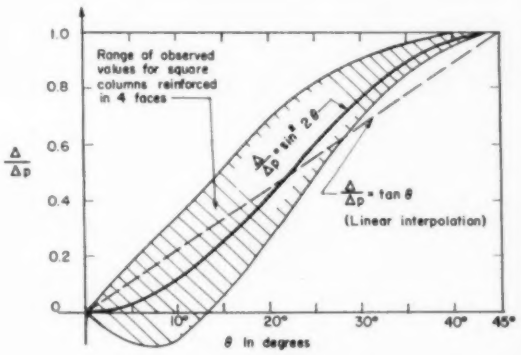

Fig. A—Deviation ratios for various skew angles

cent were observed, but generally variations were closer to 1 to 3 percent. The equation decreases in accuracy as the ratio f_c'/pf_y increases.

Mr. Pannell's suggested modifications to Conclusions 1, 3, and 4 are correct. However, for high values of P/P_0, the ratio of e'/t is less than 0.10, and for very low values of P/P_0, the reinforcement quotient would rarely be low, so that, excluding the four-bar column, for practical cases Conclusions 1 and 3 are valid. The four-bar columns were excluded from the presentation. The author at no time advocated the use of the equation presented in Conclusion 4, but recommended the use of a modified elliptical formula which is much more accurate than the existing Eq. (19) of the ACI Building Code,[10] which implies the use of a rectangular contour line for interaction surfaces. Reference 6, by Mr. Pannell, presents an adequate and reasonably simple formulation of a design technique, and his procedures or the equations of Bresler[8] are the best available. Such formulations together with the graphic description of specific interaction surfaces presented by the author will help the experienced designer to a rapid and safe solution of problems involving biaxially eccentric column loads.

REFERENCES

10. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-56)," ACI JOURNAL, *Proceedings* V. 52, No. 9, May 1956, pp. 913-986.